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Convergence of fracture process zone size in cohesive zone modeling ☆

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Nonlinear fracture process zone is associated with various material failure mechanisms, and thus its size estimation is of fundamental issues in understanding material failure behaviors. Then, the size of the fracture process zone is computationally estimated by utilizing a cohesive zone modeling approach. Geometrically similar single edge notched bending and compact tension configurations are employed with various combinations of the fracture energy, cohesive strength and elastic modulus, which lead to 91 cases. The computational results demonstrate the consistency and convergence of the fracture process zone size according to the change of the material properties and the increase of structural sizes. Additionally, the fracture process zone size is nondimensionalized through using a characteristic length. The nondimensionalized results illustrate the independence of material properties and structural geometries according to the increase of structural sizes. Therefore, the fracture process zone size in the cohesive zone model can be considered as an intrinsic material property.

1. Introduction

The estimation of fracture process zone is of fundamental issues in understanding nonlinear material failure behaviors. The fracture process zone is generally resulted from various mechanisms such as void nucleation, intergranular fracture, crack shielding due to micro-cracks, crack deflection, aggregate bridging, crack blunting, etc. Then, the size of the fracture process zone was analytically approximated in conjunction with linear elastic fracture mechanics, as summarized in Table 1. Under the assumption of a constant stress redistribution ahead of a crack tip, a plastic zone size was evaluated by satisfying an equilibrium condition [1] or relating stress intensity factors from remote tension to closure stresses at a crack tip [2,3]. Hillerborg et al. [4] addressed the effects of a critical length on concrete fracture behavior. In order to account for fracture of a progressively softening material like concrete, Bazant and Planas [5] employed a parabolic shape with degree n for stress distribution ahead of a crack tip. Hui et al. [6] estimated the cohesive zone length for a soft elastic solid by equating a stress field to a characteristic chain fracture stress. Note that although various approximation approaches are employed for the estimation of the fracture process zone size, all the estimated sizes are proportional to a characteristic size, defined as

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Table 1
Analytical approximation of the fracture process zone size.

Fracture process zone	References
$\frac{1}{\pi}EG_F/\sigma_{\max}$	Irwin [1]
$\frac{\pi}{8}EG_F/\sigma_{\max}$	Barenblatt [2] and Dugdale [3]
$EG_F/\sigma_{\max}$	Hillerborg et al. [4]
$\frac{n+1}{\pi}EG_F/\sigma_{\max}$	Bazant and Planas [5]
$\frac{2}{3\pi}EG_F/\sigma_{\max}$	Hui et al. [6]

$$\ell_{ch} = \frac{EG_F}{\sigma_{\max}^2}, \tag{1}$$

where E, G_F and $\sigma_{\max}$ are the elastic modulus, the fracture energy and the cohesive strength, respectively.

Alternatively, the fracture process zone was measured by utilizing various experimental techniques. For example, dye penetration was used to estimate the position and shape of a crack front [7,8]. Acoustic emission waves resulted from microcracking were detected by acoustic sensors, and related to the formation of the fracture process zone [9,10]. Additionally, a digital image correlation technique was employed to investigate the size and width of the fracture process zone and extract corresponding fracture parameters [11–13]. Note that the measured fracture process zone size depends on specimen geometry, structural sizes and material properties. Furthermore, it is generally different from the analytically estimated fracture process zone size listed in Table 1.

In this context, the fracture process zone size is computationally estimated by utilizing a cohesive zone modeling approach. The present study demonstrates the consistency and convergence of the fracture process zone size in the cohesive zone model, and thus the process zone size can be considered as an intrinsic material property. The effects of material properties and structural sizes on the fracture process zone are also addressed. The remainder of the paper is organized as follows. Section 2 presents the basic concept and finite element formulation of a cohesive zone modeling for crack growth simulation. Then, the fracture process zone in the cohesive zone model is defined and estimated in Section 3. In Section 4, computational results of single edge notched bending and compact tension test are provided with various combinations of material properties and structural sizes. Finally, the key findings are summarized in Section 5.

2. Cohesive zone modeling

Nonlinear fracture process zone is approximated by utilizing the concept of a cohesive zone model [2,3]. The cohesive zone model defines a traction–separation relationship in order to account for progressive damage and fracture mechanisms along fracture surface. For example, Barenblatt [2] and Dugdale [3] assumed a constant stress distribution for elastic–plastic fracture, while Hillerborg et al. [4] utilized a linear softening model for crack growth analysis of concrete. Further indepth reviews on traction–separation relations can be found in literature (e.g., [14]).

In the standard finite element method, the cohesive zone model is generally implemented by introducing cohesive surface elements. Prior to computational simulation, cohesive surface elements are inserted between continuum elements within a potential crack path, which leads to an *intrinsic cohesive zone modeling* approach. The intrinsic cohesive zone modeling approach has been widely utilized to investigate various failure behaviors such as quasi-brittle materials [15,16], adhesive bond joints [17,18], composite materials [19,20], etc. Alternatively, the cohesive zone model can be represented by other computational approaches such as extrinsic cohesive zone models [21–23], generalized/extended finite element methods [24], meshless methods [25], virtual internal bond models [26,27], peri-dynamics [28], etc. Note that the intrinsic cohesive zone model approach is the choice of the present study.

The finite element formulation of the intrinsic cohesive zone model is obtained from the principle of virtual work. The virtual work done by the external traction ($\boldsymbol{T}_{ext}$) on boundary (Γ) is equal to the summation of the virtual strain energy in domain (Ω) and the virtual cohesive fracture energy on fracture surface (Γ_c)

$$\int_\Omega \delta\epsilon : \boldsymbol{\sigma} dV + \int_{\Gamma_c} \delta\boldsymbol{\Delta} \cdot \boldsymbol{T}_c dS = \int_\Gamma \delta\boldsymbol{u} \cdot \boldsymbol{T}_{ext} dS, \tag{2}$$

where $\delta\varepsilon$ and $\delta\boldsymbol{u}$ are the virtual strain and the virtual displacement, respectively, and $\boldsymbol{\sigma}$ is the Cauchy stress tensor. The virtual cohesive fracture energy is obtained from the dot product of the virtual cohesive separation ($\delta\boldsymbol{\Delta}$) and the cohesive traction ($\boldsymbol{T}_c$) where the cohesive traction–separation relationship is embedded in cohesive surface elements. Then, the internal force vector ($\boldsymbol{f}_{coh}$) and the tangent matrix ($\boldsymbol{K}_{coh}$) of cohesive surface elements are given as

$$\boldsymbol{f}_{coh} = \int_{\Gamma_c} \boldsymbol{B}_c^T \boldsymbol{T}_c dS, \tag{3}$$

and

$$\boldsymbol{K}_{coh} = \int_{\Gamma_c} \boldsymbol{B}_c^T \frac{\partial \boldsymbol{T}_c}{\partial \boldsymbol{\Delta}} \boldsymbol{B}_c dS, \tag{4}$$

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