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Infinite mode analysis of a general model with external harmonic excitation

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ABSTRACT

This study proposes a general solution procedure for infinite mode analysis. The equation of motion is written in a general form using spatial differential operators, which are suitable for perturbation techniques. The multiple time scales method is applied directly to solve the proposed equation of motion. General investigations of some resonance cases are provided, such as parametric, sum type, difference type, and a combination of sum and difference type resonances. The proposed general solution procedure is applied to one- and two-dimensional problems. The results demonstrate that this general solution procedure obtains good solutions in the dynamic analysis of beams, plates, and other structures.

1. Introduction

Partial differential equations are used for the dynamic modeling of structures such as beams, plates, and shells. The dynamic responses of the free and forced systems are analyzed by solving these equations using an appropriate method. Perturbation methods such as straightforward, Lindstedt–Poincaré, renormalization, and multiple time scales are used widely to obtain approximate solutions during the dynamic modeling of structures. In particular, the method of multiple scales is used very frequently. The method of multiple scales can be used in two different ways. First, it can be applied directly to the partial differential equations and boundary conditions, which is known as the direct-perturbation method. Second, the partial differential equations are discretized initially using the Galerkin method before applying the method of multiple scales to the set of ordinary differential equations that is obtained, which is known as the discretization–perturbation method. Previous studies have compared the application of the direct approach and the discretization approach to the analysis of vibrations of continuous systems in general models [1–3]. These studies demonstrated that the direct-perturbation method and the discretization–perturbation method exhibit identical performance for infinite modes, whereas the direct-perturbation method yields better results for finite mode truncations. However, it is impractical to consider an infinite number of modes in a specific problem for a continuous system. Even if we assume that an infinite number of modes are available, it is easier and more convenient to use the direct-perturbation method rather than the discretization–perturbation method. In this study, the direct multiple times scales method is employed for the infinite mode analysis of a dynamic model in general form. The dynamic analysis of continuous media with a finite number of modes is generalized to an analysis that can be applied to infinite number modes.

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The dynamics of engineering models are described in the form of partial differential equations. However, instead of solving each model, general solution procedures can be developed. Thus, Pakdemirli [1] first developed a new operator notation that is suitable for perturbation calculations. This operator notation for continuous systems was extended by Pakdemirli et al. [4–8], Lacarbonara [3], and Nayfeh [9]. In these studies, a general model with quadratic and cubic nonlinearities was analyzed for a forced system or a parametrically excited system. Comparisons of direct- and discretization–perturbation methods, investigations of primary and parametrically resonance cases, and the analyses of some internal resonance cases, such as one to one, two to one, and three to one, were also presented. Mahmoodi et al. [10] studied more general operators, including cubic inertia, damping, and stiffness nonlinearities, as well as investigating the nonlinear mode shapes and natural frequencies.

Other problems examined by Ghayesh et al. [11,12], which are formulations of motion (a physical model of a mathematical model), yield a set of nonlinear partial differential equations with nonlinear, time-dependent, and coupled internal boundary conditions.

In addition, gyroscopic and Coriolis effects play important roles in rotating structures, such as rotors, spinning beams, and rotating beams, as well as in axially moving continua, such as pipes/strings, conveying fluids, and axially moving plates/beams/strings. Özhan and Pakdemirli [13–15] and Hosseini and Zamanian [16] investigated the dynamics of general continuous systems with gyroscopic and Coriolis effects. A linear general model with fractional order damping [17] and some applications [18] were also analyzed for primary and parametric resonance cases.

Pakdemirli and Öz [19] investigated the infinite mode analysis of an axially accelerated beam, which we extend to a general form in the present study. The general model is improved to allow the dynamic analysis of visco-elastic beams and gyroscopic systems. Thus, two very similar linear operators that include time derivatives are defined in the proposed model. The method of multiple scales is applied directly to analyze the infinite mode interactions of vibrations in the general model. Using this method, we determine the frequencies, steady-state primary response, and stability conditions, and we analyze some resonance cases, such as sub- and super-harmonic resonances, some internal resonances, and parametric and primary resonances. We investigate principal parametric resonances, sum and difference type resonances, and further interactions that involve up to four different modes of the general model.

2. Equations of motion

We consider a fairly general class of continuous systems in a dimensionless form as

$$L_1(\ddot{w}) + L_2(\dot{w}) + L_3(w) + \varepsilon\{L_4(w) + L_5(\dot{w}) + (L_6(w) + L_7(\dot{w}))(e^{\Omega t} + cc)\} = 0, \tag{1}$$

where cc is the complex conjugate of the former term and ε is a small dimensionless physical parameter. The overdot indicates differentiation with respect to the non-dimensional time t. There may be more than one spatial variable and a three-dimensional problem that involves spatial variables x, y, and z is not excluded from the analysis. w represents dimensionless deflection. L_j are linear and homogeneous, self-adjoint, and positive-definite differential or integral–differential operators. L_1 corresponds to the inertia force. L_2 is used to express the circularity effects of the system that act on $\dot{w}$ in a manner such that the system remains conservative. L_3 is related to the stiffness of the system. L_4 and L_5 denote damping forces and the system becomes non-conservative with these operators. The operators L_6 and L_7 with the accompanying harmonic term represent a general parametric excitation. Ω represents the parametric excitation frequency. Note that the proposed linear model is fairly general and any linear vibration problem, including visco-elastic materials, can be cast into the formalism of Eq. (1), and the dynamic analysis obtained can be approximated using the algorithm developed in the following sections.

A restriction on the boundary value problem comes from the boundary conditions. The boundary conditions are indicated as:

$$B_{11}(w) = B_{12}(w) = 0, \tag{2.a}$$

$$B_{21}(w) = B_{22}(w) = 0. \tag{2.b}$$

B_{ij} are self-adjoint, linear operators of boundary conditions. j denotes the jth condition of the ith support. Note that both the equations of motion and the boundary conditions should be expressed in non-dimensional form for applications.

3. Direct treatment of the general model

The method of multiple scales is applied directly to determine a second-order uniform expansion of the solution of Eqs. (1) and (2) when the system is subject to a parametric resonance of the nth mode and internal resonances are engaged. Hence, a solution of Eqs. (1) and (2) is sought in the following form:

$$w = w_0 + \varepsilon w_1 + O(\varepsilon^2), \tag{3}$$

where $T_0 = t$ is the usual fast time scale and $T_1 = \varepsilon t$ is the slow time scale. The time derivatives are expressed as:

$$\frac{d}{dt} = D_0 + \varepsilon D_1 + O(\varepsilon^2), \tag{4.a}$$

$$\frac{d^2}{dt^2} = D_0^2 + 2\varepsilon D_0 D_1 + O(\varepsilon^2). \tag{4.b}$$

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