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An optimization model for products with limited production quantity

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ABSTRACT

An important strategy for dealing with scarcity and customer response is to produce limited quantity of certain products. Due to limited production quantity, consumers would feel the value or uniqueness of the products and have a stronger urgency to purchase them. Some distribution outlet would raise prices to cover promotion expenses and to increase profit margin. In this study, we consider a newsvendor problem for products with limited production quantity: both the unit selling price and customers' demand are influenced by the limited production quantity. An algorithm is developed to obtain a production policy such that the expected profit is maximized. Numerical examples and sensitivity analysis are presented to illustrate the model.

1. Introduction

Scarcity of matter is a pervasive aspect of human life and is the fundamental precondition of economic behavior [1]. Consumers often consider possessing scarce products to show their uniqueness, and it triggers them to desire these products. Consequently, some manufacturers may design their marketing strategies by producing a limited quantity of their products. This type of marketing strategy is very common among innovative products. For example, department stores announce limited products on their promotional flyers during their anniversary sales. Customers are required to book in advance or wait in line in order to buy the limited products. Besides, it is very common for some famous product brands to offer limited products during their seasonal sales to attract attention and sales. For another example, Motorola V3i cooperated with D&G to launch a limited version of luxury cellphone called Color Gold; they only produce 1000 pieces of such version worldwide, and its selling price is much higher than other unlimited production quantity version [2].

Commodity theory [1,2] deals with the psychological effects of scarcity. The theory claims that "any commodity will be valued if it is unavailable". According to the theory, scarcity enhances the value (or desirability), and it gives to its possessor a sense of pride in possessing the limited product [1]. The feeling of uniqueness may vary for different situations and persons; as such, it may be related to: (a) forces in a given situation that promote an extreme sense of high similarity, and (b) dispositional factors that influence the high need for uniqueness across a variety of situations [3]. Sirgy [4] addressed the importance of scarcity in marketing strategy. Salespersons should apply such strategy while merchandising products or services; it will increase the motivation of the targeted customers to approach the promotional information. There are two strategies for

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price raise through scarcity: (1) direct result from the quality and symbolic interest, and (2) indirect result on quality and symbolic interest through the price. As a result, raising the prices of scarce products can make a positive impact, but also may backfire if it is not launch properly [5]. Therefore, if we combine the commodity theory and the need for uniqueness theory, we can demonstrate that customers prefer possessing scarce product to show their uniqueness, compared to possessing common and easily-available products.

Recently, many industries apply the strategy of limited production through single production schedule. In inventory management, this problem is known as the "newsboy problem" or the "newsvendor problem". The newsboy problem is a single period stochastic inventory problem [6,7] which deals with stocking issues in today's supply chains [8–10]. Weng [11] analyzed the coordinated quantity decisions between the manufacturer and the buyer in a newsvendor model. Dominey and Hill [12] explored the effectiveness of approximating a compound Poisson distribution in a newsboy model. Wang and Webster [13] used loss aversion to model manager's decision-making behavior in the single-period newsvendor problem. Shi et al. [14] extended the multi-product newsvendor problem by incorporating the retailer's pricing decision considering supplier quantity discount. From our literature search, no researches have been done on the newsvendor problem to consider the limited production quantity issues.

In this study, the supplier has to consider the uncertainty in customer demand. Having a good manufacturing and marketing strategy of the limited-edition products before the selling period of the product is vital to the supplier. We present an algorithm to derive an optimal production quantity and selling price such that the expected profit is maximized.

2. Notations

The following notations are used in our analysis:

$E\pi$	the expected profit for the supplier
Q	the production quantity for the supplier; decision variable
Q^*	the optimal production quantity for the supplier considering limited production quantity
Q_w	the production quantity for the supplier without considering limited production quantity
Q_w^*	the optimal production quantity for the supplier without considering limited production quantity
p_1	the selling price per unit without considering limited production quantity; constant
p_2	the upper bound of selling price per unit when the production quantity is limited; constant
$p(Q)$	the selling price per unit with considering limited production quantity; which is a function of production quantity
c_p	the production price per unit; $c_p < p(Q)$
s	the salvage value per unit $s < c_p$
r	the shortage cost per unit; represents costs of lost goodwill
x	the random demand with the PDF (Probability Density Function), $f(x)$, and CDF (Cumulative Distribution Function), $F(x)$

3. Modeling and assumptions

Throughout this study, single production of the limited product is assumed. The supplier manufactures a batch of the products, Q, and sells to the retailer or directly to the customers. The unit production price of the product is c_p. For simplicity, the unit production cost is assumed to be constant. The unit selling price is $p(Q)$. When the sale quantity is less than the batch Q, the leftover is sold with a unit salvage value s. When the demand is more than the batch, Q, shortage occurs. All shortages will be lost sale and the unit lost sale shortage cost is r. For the selling price $p(Q) = p_1$, the supplier will manufacture an optimal batch of Q_w^*. This is identical to the newsboy problem. The suppliers' expected profit function $E\pi$ is:

$$E\pi(Q_w) = \int_0^{Q_w} \{[p_1 - c_p]x - (c_p - s)(Q_w - x)\} f(x)dx + \int_{Q_w}^{\infty} \{[p_1 - c_p]Q_w - (x - Q_w)r\} f(x)dx. \tag{1}$$

Similar to Hadley and Whitin [15], the suppliers' optimal production batch is:

$$F(Q_w^*) = (p_1 - c_p + r)/(p_1 - s + r), \tag{2}$$

where $F(x)$ is the CDF of x. If the supplier manages the limited production batch, then the consumers' perceived value and purchase decisions are usually influenced by the law of scarcity [1]. The unit selling price $p(Q)$ of the limited quantity products is a decreasing function of Q. However, the customer demand will decrease due to a higher selling price. That is, the random demand of the products depends on production batch, Q, because the higher production batch will decrease the selling price, while the lower selling price will increase demand. That means the PDF, $f(x)$, of the random demand x is a function of Q. The suppliers' expected profit function $E\pi$ is given as follows:

$$E\pi(Q) = \int_0^{Q} \{[p(Q) - c_p]x - (c_p - s)(Q - x)\} f(x)dx + \int_{Q}^{\infty} \{[p(Q) - c_p]Q - (x - Q)r\} f(x)dx. \tag{3}$$