Applied Mathematical Modelling 39 (2015) 1929–1940

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

A general model for the exact computation of yield from a rainwater tank

John Mashford *, Shiroma Maheepala

Commonwealth Scientific and Industrial Research Organisation (CSIRO), PO Box 56, Highett, Vic. 3190, Australia

ARTICLE INFO

Article history:
Received 30 August 2012
Received in revised form 28 January 2014
Accepted 2 October 2014
Available online 18 October 2014

Keywords:
Rainwater tank
Simulation
Continuous time
Exact

ABSTRACT

The models presented in the literature for the computation by simulation of yield from a rainwater tank are either incorrect or approximate at best. In this paper a general, exact, continuous time model of the operation of rainwater tanks is presented. The operation of a tank is determined by the rainfall runoff and demand distribution functions and the capacity of the tank. The runoff and demand distributions are assumed to satisfy certain reasonable conditions which are satisfied in all cases of practical interest, e.g. when the distributions are described by piecewise constant functions. The yield, spill and volume functions are characterized by equations which, by means of a constructive proof involving mathematical induction, can be shown to have unique solutions. This model can be used to compute the exact yield and spill over any period and the volume at any time during the operation of the tank. Correct discrete time simulation models are derived in the special cases of demand before rain (DBR) or demand after rain (DAR).

1. Introduction

Rainwater tanks provide an additional source of water to traditional centralized urban water systems. Also they provide benefits in terms of downstream stormwater infrastructure requirements and pollutant load reduction [1,2].

A rainwater tank forms a particularly simple type of system. It consists of a container of some fixed capacity and it has a number of inputs and outputs which can be represented by the following mass balance equation [2].

$$V_{end} = V_{start} + R + P - E - S - L - Y, \tag{1}$$

where V_{start} and V_{end} are the volumes of water in the tank at the start and end respectively of a given time period under consideration, R is the rainfall runoff from the roof of the house to which the rainwater tank is connected, P is the precipitation incident on the rainwater tank, E is the evaporation from the rainwater tank, S is the amount of spillage from the rainwater tank due to overflow, L is the seepage (or leakage) losses and Y is the yield of the tank to the householder. In the case of a covered tank the incident precipitation and evaporation terms can be assumed to be zero. We will assume, as is usually assumed by researchers in this area, that the rainwater tank is covered and also that the leakage losses can be neglected. Thus the mass balance equation that we are assuming is

$$V_{end} = V_{start} + R - S - Y. \tag{2}$$

* Corresponding author. Tel.: +61 3 9252 6115; fax: +61 3 9252 6253.
E-mail addresses: John.Mashford@csiro.au (J. Mashford), Shiroma.Maheepala@csiro.au (S. Maheepala).

http://dx.doi.org/10.1016/j.apm.2014.10.004

Spillage occurs when the storage capacity is exceeded and yield occurs when there is household water demand and also there is water in the rainwater tank. Demand which is not met by water from the rainwater tank is met by the mains water supply.

The volume of water in the rainwater tank and the total yield from the tank vary with time and are dependant on the distributions of rainfall runoff and demand and the capacity of the tank. The behavior of a rainwater tank for any given rainfall and demand data, roof runoff model and tank capacity may be estimated by so called behavior analysis involving discrete time simulation [2–6].

Simulation of the operation of rainwater tanks is important for obtaining yield estimates to determine the effect of rainwater tanks on a regional water supply [7] and determining the optimal volume of a rainwater tank [8]. The effect of a large number of rainwater tanks can be estimated by stochastic simulation [9]. The key component of such analyses is the simulation of the operation of a single rainwater tank.

Discrete simulation models for this purpose have been described by Fewkes [4], Liaw and Tsai [5] and others. Unfortunately, however, their models are incorrect, as is explained below. This casts some doubt onto the results of their work.

Consider the operation of a rainwater tank over T time periods where $T \in \{1, 2, \ldots\}$. Let, for $t \in \{1, \ldots, T\}$,

R_t = rainwater runoff during time period t.
V_t = volume of water in tank at the end of time period t.
Y_t = yield from tank during time period t.
D_t = demand for water during time period t.
C = tank capacity.

The rainfall runoff time series is usually estimated from rainfall data together with a rainfall runoff model involving a number of parameters including roof area while the demand time series is usually estimated using a demand model [7].

We may derive valid discrete simulation equations in the special case where, in each time period, the demand comes before the rainfall or the demand comes after the rainfall as follows.

1.1. Demand before rain (DBR)

Water Y_t going towards meeting the demand D_t is withdrawn from the tank at the beginning of time period t after which the runoff R_t is added with spill if the tank fills up to a volume C. At the beginning of time period t the volume of water in the tank is V_{t-1}. If the demand D_t is less than or equal to V_{t-1} then the yield Y_t will be D_t. If the demand is greater than V_{t-1} then the yield Y_t will be V_{t-1} (and the rest of the demand will be met from the mains supply). Thus

$$Y_t = \min(D_t, V_{t-1}). \tag{3}$$

Now, after Y_t has been withdrawn from the tank R_t is added. As soon as the volume of water in the tank reaches C, spill occurs. Therefore the volume of water in the tank at the end of the t^{th} time period is

$$V_t = \min(V_{t-1} - Y_t + R_t, C). \tag{4}$$

1.2. Demand after rain (DAR)

Water from rainwater runoff R_t is added to the tank at the beginning of time period t with spill if the volume of water reaches C after which the water Y_t going towards meeting the demand D_t is withdrawn from the tank. At the beginning of time period t the volume of water in the tank is V_{t-1}. Therefore, after R_t is added, the volume of water in the tank is $\min(V_{t-1} + R_t, C)$. Hence, the water Y_t withdrawn from the tank is

$$\begin{aligned} Y_t &= \min(\min(V_{t-1} + R_t, C), D_t) \\ &= \min(V_{t-1} + R_t, C, D_t). \end{aligned} \tag{5}$$

Thus the volume of water in the tank at the end of time period t is

$$V_t = \min(V_{t-1} + R_t, C) - Y_t. \tag{6}$$

1.3. Yield before spill (YBS), yield after spill (YAS) as presented by Fewkes, Liaw and Tsai and others

Now DBR is a special case of yield before spill (YBS) and DAR is a special case of yield after spill (YAS).

In YAS rainfall and possible spill may occur before there is any yield. Fewkes' and Liaw and Tsai's equations for this case are

$$Y_t = \min(V_{t-1}, D_t), \tag{7}$$

$$V_t = \min(V_{t-1} + R_t - Y_t, C - Y_t). \tag{8}$$