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High-precision single-input control of relative motion in spacecraft formation

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ABSTRACT

A Newton-type method is proposed to improve the accuracy of control for relative motion of two satellites in close formation. We assume that the deputy satellite is equipped with a passive attitude control system that provides one-axis stabilization, and one or two orbit control thrusters are installed along the stabilized axis. Previous studies show that it is possible to construct periodic relative trajectories both in case of passive magnetic and spin stabilization. However, the accuracy of the numerically obtained control is quite low due to modeling errors caused by linearization of the equations of relative motion. Therefore, a correction procedure is required to compensate for non-linear effects. To this end we suggest a recently developed algorithm based on the Newton method for solving nonlinear systems with geometric constraints. Being implemented, this algorithm allows decreasing the modeling error by up to ten times. The previously found control and trajectory of the linearized system are used as initial approximations.

0. Introduction

Inexpensive small satellites play a continuously growing role in the modern era of space exploration. Several small satellites flying in formation are able to perform quite elaborate missions while keeping program budgets within reasonable limits [1,2]; however, control of their relative motion represents a challenge [3–7]. As a result, more and more studies investigate the ability to control spacecraft motion in compliance with typical limitations of small satellites. One of the major issues here is that such satellites often do not possess a complex attitude control system, so three-axis stabilization might be unavailable and the thrust vector of the orbit control system cannot be arbitrary oriented in space. Quite often the option is to use a simple and lightweight passive system of one-axis stabilization, such as spin stabilization or passive magnetic stabilization. In this case, one or two (in opposite directions) orbit control thrusters can be installed along the stabilized axis, so the orientation of the thrust vector at any given moment in time is determined by the orientation of stabilized axis. This kind of control is usually referred to as single-input control since only the scalar magnitude of control acceleration can be changed.

Applications of the single-input control concept to the problem of formation keeping have been considered for

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several missions (see, e.g., Refs. [8–10]). Much research is focused on compensation for the satellites' relative drift caused by the J_2 harmonic of the Earth's gravitational potential. In our previous paper [8], a possibility to obtain a periodic relative motion of the chief and deputy satellites has been demonstrated for several types of single-input control, including the control oriented along the geomagnetic field and the control along an axis fixed in absolute space. In each case the sufficient controllability conditions have been deduced. These conditions can be concisely formulated as follows: the vector of control direction should have non-zero components both in the orbital plane and along the normal to the orbit. To describe the relative motion of formation flying spacecraft, the Schweighart–Sedwick modification of the Hill–Clohessy–Wiltshire equations has been used.

In the present paper we complete the method of Ref. [8] with a Newton-type method that allows one to correct the trajectory by improving the control law obtained from the linearized model. Implementation of this iterative procedure significantly reduces the modeling error and validates the results theoretically proved for the linearized system.

1. Linearized equations of relative motion

Consider a system of two satellites, chief and deputy, moving in close near-circular orbits. Without loss of generality, we assume that the chief satellite moves passively while the deputy one is controlled. We introduce the reference circular orbit which is close to the orbits of both satellites. Denote the inclinations of the reference orbit and of the orbits of chief and deputy satellites by $i_{\mathtt{ref}}$, i_1, and i_2, respectively. Later on we use the following LVLH reference frame $Oxyz$ with its origin at the chief satellite: Oz-axis lies in the radial direction outwards from the Earth, Oy is normal to the orbital plane, and Ox completes the right-hand frame.

To describe the relative motion, we use the Schweighart–Sedwick (SS) modification of the Hill–Clohessy–Wiltshire (HCW) equations [11]

$$\ddot{x}+2nc\dot{z}=w(t)e_x(t),$$
$$\ddot{y}+q^2y=2lq\cos(qt+\phi)+w(t)e_y(t),$$
$$\ddot{z}-2nc\dot{x}-(5c^2-2)n^2z=w(t)e_z(t).$$

Here x, y, and z are the coordinates of the deputy satellite in $Oxyz$ frame and

$$q=nc+\frac{3nJ_2R_\oplus^2}{2r_{\mathtt{ref}}^2}\left(\cos^2 i_2-\frac{(\cos i_1-\cos i_2)(\cot i_1\sin i_2\cos\Delta\Omega_0-\cos i_2)}{\sin^2\Delta\Omega_0+(\cot i_1\sin i_2-\cos i_2\cos\Delta\Omega_0)^2}\right),$$

$$l=-\frac{3nJ_2R_\oplus^2}{2r_{\mathtt{ref}}}\frac{(\cos i_1-\cos i_2)\sin i_1\sin i_2\sin\Delta\Omega_0}{\sqrt{1-(\cos i_1\cos i_2+\sin i_1\sin i_2\cos\Delta\Omega_0)^2}},$$

$$c=\sqrt{1+s},\quad s=\frac{3J_2R_\oplus^2}{8r_{\mathtt{ref}}^2}(1+3\cos 2i_{\mathtt{ref}}),$$

$$\Delta\Omega_0=\frac{y_0}{r_{\mathtt{ref}}\sin i_{\mathtt{ref}}},\quad y_0=y(0)$$

where n and $r_{\mathtt{ref}}$ are respectively the mean motion and the radius of the reference orbit, $R_\oplus$ is the Earth's equatorial radius, and $J_2\approx 10^{-3}$ is the second zonal harmonic. The phase ϕ can be determined from the equation

$$l\sin\phi+qy_0\cot\phi=\dot{y}_0,$$

where $\dot{y}_0=\dot{y}(0)$. The direction of the control acceleration $w(t)$ is defined by the vector-function

$$\boldsymbol{e}(t)=(e_x(t),e_y(t),e_z(t))^T.$$

Both the HCW equations and the SS equations are linearized about the reference circular orbit. However, the latter take into account the influence of the J_2 harmonic of the Earth's geopotential, and hence they describe the relative motion more precisely.

2. Closed trajectories of linearized system

Using the notations

$$\boldsymbol{\eta}=(x,y,z,\dot{x},\dot{y},\dot{z})^T,$$
$$\boldsymbol{a}(t)=(0,0,0,0,2lq\cos(qt+\phi),0)^T,$$
$$\boldsymbol{b}(t)=(0,0,0,e_x(t),e_y(t),e_z(t))^T,$$
$$A=\begin{pmatrix}0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\0&0&0&0&0&-2nc\\0&-q^2&0&0&0&0\\0&0&(5c^2-2)n^2&2nc&0&0\end{pmatrix},$$

one obtains a canonical form of linear single-input control system

$$\dot{\boldsymbol{\eta}}(t)=A\boldsymbol{\eta}(t)+\boldsymbol{a}(t)+\boldsymbol{b}(t)w(t),\quad \boldsymbol{\eta}(t)\in R^6. \tag{1}$$

We say that system (1) has a T-closed trajectory $\boldsymbol{\eta}(\cdot)$ satisfying $\boldsymbol{\eta}(0)=\boldsymbol{\eta}_0$, if and only if there exists an admissible control $w_{\boldsymbol{\eta}_0}(\cdot)$ such that

$$\boldsymbol{\eta}_0=e^{TA}\boldsymbol{\eta}_0+\int_0^T e^{(T-t)A}(\boldsymbol{a}(t)+w_{\boldsymbol{\eta}_0}(t)\boldsymbol{b}(t))\,dt. \tag{2}$$

In [8] three different types of single-input control are considered. For the bilateral control ($w(t)\in R$) oriented along either a vector fixed in the inertial space (the case of spin stabilization) or the vector of local geomagnetic field (the case of passive magnetic stabilization), it was shown that, under some nonrestrictive conditions, for any initial point and any $T>0$ there exists a T-closed trajectory of system (1). Therefore, this control ensures T-periodic relative motion of satellites.

For the unilateral control ($w(t)\geq 0$), the situation is more involved. In the case of passive magnetic stabilization, the function $\boldsymbol{b}(t)$ is τ-periodic, with $\tau=2\pi/(nc)$, and for any initial point there exists an $M\tau$-closed trajectory of system (1) where M is some natural number. The value of M can be determined in the following way. Let $\boldsymbol{\Xi}$ be some simplex in R^6 containing the origin as an interior point. Consecutively taking its vertices ξ_k, $k=1,\dots,7$, as initial points and numerically solving (2), one can obtain minimal natural numbers M_k, $k=1,\dots,7$, such that for ξ_k there exists an $M_k\tau$-closed trajectory. Then $M=\max\{M_k|k=1,\dots,7\}$. One can show [8] that M=2.