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Dependence of noise induced effects on state preparation in multiqubit systems

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ABSTRACT

The perturbation of multiqubit systems by an external noise can induce various effects like decoherence, stochastic resonance and anti-resonance, and noise-shielding. We investigate how the appearance of these effects on disentanglement time depends on the initial preparation of the systems. We present results for 2-, 3- and 4-qubit chains in various arrangements and observe a clear dependence on the combination of initial geometry of the state space and the placement of noise. Finally, we see that temperature can play a constructive role for the control of these noise induced effects.

1. Introduction

Multiqubit systems are the building blocks for the architecture of quantum information storing and processing [1]. Their efficiency depends strongly on the possibility to retain quantum coherence. But these quantum systems are physically embedded and constantly interacting with their local quantum and classical environments which affect coherence [2]. Thus being fundamentally open quantum systems, they dissipate and decohere with time constants that depend on their parameters and coupling constants and on the strength of perturbation by external agents. There have been many efforts to increase the decoherence time, either passively by increasing isolation or engineering the existence of decoherence free subspaces [3] or actively by affecting the dynamics with external controls [4–6].

As it has turned out, a counterintuitive and unexpected external perturbation with a positive role is the action by a noise source, quantum or classical. Normally one expects that noise would only increase decoherence in a monotonous way, namely increased destruction of coherence with higher levels of noise. But it has been found that noise may be used to isolate a quantum system, the noise-shielding effect (N.S.), and may influence positively or negatively the decoherence time in a monotonous or non-monotonous way. These are the quantum Zeno effects and the stochastic resonance (S.R.) or stochastic anti-resonance (S.AR.) effects [7–13]. But it appears that the manifestation of these effects depends, apart from the detailed dynamical setup, on the way the system has been prepared.

From our preliminary investigation of noise effects on two-qubit systems [13], it became evident that certain classes of initial states are more sensitive than others. Considering this not to be accidental, we extended our study of the state preparation dependence of the noisy perturbations, to 3- and 4-qubit Heisenberg XY chains. We have found a clear correlation between the way a quantum system of qubits reacts to an external classical noise and the geometry of the initial states. As a tool we used a Master equation for modeling the dissipative and decoherence processes, and applied an external classical Gaussian white noise as a stochastic control. Our investigation is based on the decoherence properties of a 2-qubit subsystem. In the case of two qubits we have only the bosonic environment and the external classical noise. In the cases of 3- and 4-qubit systems we consider the extra qubits, after they are traced out, as a form of local fermionic environment and quantify the entanglement with the concurrence of the 2-qubit subsystem. We study numerically the dependence of disentanglement time (or entanglement sudden death (E.S.D.)) [14–21] on the strength of the applied noise for zero and non-zero temperature and for various initial states. We have observed the effects of increase of decoherence, stochastic resonance, stochastic anti-resonance and noise-shielding.

The main result is that these effects depend strongly on the initial preparation of the compound system and the placement of classical noise. The importance of this result could be appreciated in the cases where the behavior of a studied subsystem depends on the preparation of a bigger system whose parts have

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been traced out, but they are in interaction with the subsystem of interest. Our results are summarized as follows:

2-Qubits

1. Stochastic anti-resonance is observed if the initial density matrix contains population elements $\rho_{2,2}$ or $\rho_{3,3}$. This becomes evident in Bell states $|\Psi\rangle$ (Fig. 2(a) (upper curve)), while the entanglement of Bell states $|\Phi\rangle$ decays monotonically over increasing noise strength (Fig. 2(b)).
2. For vanishing dissipation rate ($\gamma \to 0$), the Hilbert subspace which exhibits stochastic anti-resonance tends to become a decoherence free subspace [13].
3. There are no noise shields in the 2-qubit case. Noise shields require the extension of the system to more qubits, in order to apply noise on the local fermionic environment.
4. Temperature degrades stochastic anti-resonance very quickly. An average excitation number $\langle n\rangle \simeq 0.5$ is enough to eliminate the appearance of the anti-resonance (Fig. 2(a) (lower curve)).

3-Qubits

1. Multiple resonances (stochastic resonance and stochastic anti-resonance) are observed in two product states when noise affects the traced out qubit (Fig. 3(a)). Moreover, there are two product states that exhibit N.S. behavior, and two that exhibit S.AR. (Table A.1).
2. Product states with parallel spins do not present interesting behavior. Noise decreases monotonically their entanglement evolution. This holds for any arrangement of noise perturbation (Table A.1).
3. By altering the initial 3-qubit preparation of a given 2-qubit state, we observe different behavior in the 2-qubit disentanglement time. This becomes evident with the reduced 2-qubit state of a W state. A small change of initial 3-qubit preparation results in N.S., while W state preparation results in S.AR. (Fig. 4(a), (b)).
4. Bell state $|\Phi\rangle$ preparations exhibit N.S. when noise affects the traced out qubit, for all of the initial 3-qubit preparations (Fig. 3(b) and Table A.2).
5. Most of the Bell state $|\Psi\rangle$ preparations exhibit N.S. when noise affects the traced out qubit, but there is an initial system preparation which results in S.R. (Fig. 3(c) and Table A.3).

4-Qubits

1. Product states with parallel spins exhibit the same behavior as in the 3-qubit case (Table B.1).
2. Most of the product states exhibit noise shield when noise affects the traced out qubit (Fig. 5(a) and Table B.1).
3. There are four product states which exhibit different kinds of effects, depending on the placement of noise and one that exhibits S.AR. behavior when noise affects the traced out qubits (Table B.1).
4. The noise shields tend to become weaker if we increase the anisotropy of the system (Fig. 5(b)).
5. Most of the Bell state $|\Phi^+\rangle$, $|\Psi^+\rangle$ preparations continue to exhibit N.S. when noise affects the traced out qubits, except from two $|\Phi^+\rangle$ preparations that result in stochastic anti-resonance, two preparations of $|\Psi^+\rangle$ which result in S.AR., one $|\Psi^+\rangle$ preparation that exhibits multiple resonances and one $|\Psi^+\rangle$ preparation that exhibits S.R. (the last one when noise is internal) (Fig. 6(a) and Tables B.2, B.3).
6. The noise shields become stronger by applying noise in both of the traced out qubits rather than one of them (Fig. 6(b), (c)). Temperature degrades their magnitude, but does not change their shape. There is increase of tolerance against temperature compared with the 2-qubit case, because of the environmental qubits. We can observe a clear N.S. behavior with excitation values up to $\langle n\rangle \simeq 6$ (Fig. 7(a)).
7. Even though it is difficult to determine the aforementioned initial preparations which exhibit S.AR., we can control this behavior by increasing temperature. Above a critical value of the latter, S.AR. disappears and we observe N.S. This attributes an interesting positive role to the temperature, for the control of entanglement evolution (Fig. 7(b)).

A complete list of the effects is presented in the tables of Appendices A and B. In the final paragraph we classify our results in terms of the observed effects and make comments for the interplay between the geometry of the initial system preparation and the placement of noise, which seems to be crucial for their appearance.

2. XY Heisenberg model

The general form of an N-spin XY chain (for spin-1/2 particles) with nearest-neighbor interaction is

$$H_{XY} = \sum_{n=1}^{N} \left(J_x S_n^x S_{n+1}^x + J_y S_n^y S_{n+1}^y\right), \tag{1}$$

with $S_n^i = \frac{1}{2}\sigma_n^i$ $(i = x, y, z)$ the spin-1/2 operators, σ_n^i the corresponding Pauli's operators, $\hbar = 1$, $J_x \neq J_y$ and $S_{N+1} = S_1$ (periodic boundary condition). The chain is ferromagnetic if $J_i < 0$, and antiferromagnetic if $J_i > 0$. Here we study this chain in the presence of a constant external magnetic field ω_0 along the z-axis. Consequently, the unperturbed system Hamiltonian is:

$$H_0 = H_{XY} + \sum_{n=1}^{N} \omega_0 S_n^z, \tag{2}$$

for $N = 3, 4$.

The XY model has been studied for many decades because of its interesting and unusual features [22]. It is an example of integrable system. While the isotropic Heisenberg chain is solved with the use of Bethe Ansatz, the XY model is solved by means of Jordan–Wigner transformation [23] which was introduced in 1928 and applied to it by Lieb [24] in 1961. For more information see [25]. Furthermore, due to its mathematical simplicity, it is suitable for our purpose and provides a basic unit for many quantum information implementations, such as N.M.R. quantum computation and quantum teleportation [26,27].

3. Markovian Master equation

We employ a Markovian form of the Master equation, where the classical Gaussian white noise $\xi(t)$ is added to the magnetic field

$$\omega = \omega_0 + \xi(t) \tag{3}$$

and is appropriately incorporated in the double commutator [28]. For a thermal bath with $T \geqslant 0$ the Master equation is

$$\frac{d\rho_s}{dt} = -i[H_0, \rho_s] + \gamma\left(\langle n\rangle + 1\right) \sum_{n=1}^{N} \left(D\left[S_n^-\right]\rho_s\right) + \gamma\langle n\rangle \sum_{n=1}^{N} \left(D\left[S_n^+\right]\rho_s\right) - M_z\left[V_z, [V_z, \rho_s]\right], \tag{4}$$