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Modeling time and spatial variability of degradation through gamma processes for structural reliability assessment

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A B S T R A C T

The objective of this work is to propose a spatio-temporal random field for assessing the effect of spatial variability on the degradation process in structural reliability assessment. Our model extends the classical Gamma process which is usually developed for degradation temporal variability concerns to integrate spatial variability and heterogeneity issues. A log-normal distributed spatial random scale is then introduced in the Gamma process. Mathematical models for a structure degradation in space and time and estimation procedures are developed in this paper. Approximations and simulations are given to evaluate the failure time distribution and to characterize the residual lifetime after inspection. Simulation results are performed using pseudo-random data based on Monte-Carlo simulations to fit the model and the inference of their parameters. The two proposed estimation method – Method of Moments and Pseudo Maximum Likelihood – are numerically compared according to statistical measures. The accuracy of the methods are also discussed by numerical examples given the approximations of quantities of interests.

1. Introduction

In recent years, the scientific material and structural engineering community pays a lot of attention to the elaboration of mathematical degradation models for integrating spatial variability and uncertainty. It is obvious that the deterioration of structures exposed to environmental conditions is spatially and temporally varying. The variation along the space is caused by the inherent uncertainty through the material at several positions on the structure and the physical parameters involved in the deterioration mechanism. This material non–-homogeneity problem is well known for steel and concrete structures and has been, e.g., studied in [1–5] through concrete diffusion property, concrete cover and chloride external concentration.

A common way to deal with such uncertainties is to use a probabilistic framework by modeling the input data with random fields [2,10,19,21,34]. For instance, a randomized salty environment (deicing of the sea natural salts) can be integrated in the transport equation to compute the concentration of the chloride in concrete, in order to estimate the time until failure under uncertainties. In [3,5] the authors consider a two-dimensional Gaussian random field with a Gaussian correlation to compute the likelihood of corrosion-induced cracking in reinforcing steel bar. They provide an estimation of the time to first crack and time to limit crack widths. In [1], authors propose a probabilistic model for steel corrosion in reinforced concrete structures considering crack effect on the corrosion mechanism, in which an empirical model for the crack propagation stage is developed by the standard gamma process and combines corrosion crack width with steel-bar cross sectional loss. The main disadvantage of these approaches is the problem of larger dimension so called "curse of dimensionality" in which the resolution of a large number of deterministic problems is involved.

Meta-models are commonly used to tackle this problem of curse of dimensionality for degradation prediction in structural engineering. The time-dependent deterioration processes are modeled by a stochastic model where only the variation in time is studied. In particular a standard Gamma process is an appropriate mathematical model for predicting deterioration encountered in civil engineering [6,35]; such as corrosion and crack of reinforced concrete. The authors in [17,18] developed a probabilistic framework in which interaction between shocks and gradual process are combined in view to describe the deterioration process. They proposed a semi-analytic computation model to estimate with less cost the time to failure. The work in [9] describes the degradation by a Gamma process and includes other dependent-parameters as covariate in the shape function. In [26], a Gamma

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process with a random scale following a gamma law is considered to model heterogeneity in the degradation data and obtained analytic results for reliability assessment. Markov chain is another widely used approach to model cumulative damage. It is seen as a discrete Markov process where the deterioration is assumed to be a single step function. The estimation of the one-step transition matrix requires a large number of transitions to estimate all its elements [16,28]. The drawback of this approach is that time variability is difficult to capture.

In [15], the authors propose the construction of the state-dependent degradation model based on the Gamma process, where the cracking of a submerged concrete structure subjected to corrosion is described by the proposed bi-variate model with a suitable parameters. In the same time, the authors in [23] introduce a state-dependent Gamma process for the degradation, the dependency is modelled in the scale function instead of the shape function as proposed in [15]. The authors in [24] introduce a bi-variate spatio-temporal field to model the action induced by the walking of a small group of persons. Based on the spectral and coherence functions of the forces, they proposed an evaluation of vertical and transversal accelerations at nodes of a finite element.

All models based on temporal variability assume a uniform degradation and do not integrate the spatial variability through degradation process. Nevertheless, recent studies have shown that this spatial correlation has an important and direct impact on the level of structural reliability estimates [32,3,4]. Therefore, incorporating these uncertainties in the degradation processes through mathematical modelling improves their prediction and versatility in term of maintenance and decision. On the other hand, to construct an accurate model of the degradation, a large amount of data using destructive or nondestructive testing is required from a large amount of structures [31,32]. Therefore, one way of obtaining accurate and reliable information is to embed the spatial variability in the models. This allows to increase the relevance of the Meta-Model approach where the uncertainties are reduced, improving the accuracy of the inference and extending the use of the non-destructive testing.

Therefore, the major contribution of this paper is a new spatio-temporal random model based on Gamma process for predicting a single degradation measure which takes into account both temporal and spatial variability. Under the stationary assumption, the spatial monitoring data of the structure contributes in the parameters estimate to increase the accuracy of the meta-model approach. Our model requires only few parameters and is thus very suitable for inference when only few components are inspected: that is the case for on-site inspection of civil engineering structures or marine structures where the cost of inspection is high.

The spatio-temporal degradation model is assumed to be an observable process in space and time with limited observations (Non Destructive Testing, distributed sensors). However, such kind of database that considers both hazard time and space of the degradation are not available in the literature. In order to validate the proposed inference framework, we construct a synthetic discrete degradation model through Monte Carlo simulations. Therefore, numerical experiments will be conducted and compared for identifying preliminary properties and advantages of our model in terms of statistical inference and computation of quantities of interest for reliability and maintenance.

The article is organized as follows: Section 2 introduces the classical Gamma process for temporal variability, the Gaussian random field with its simulation method and the construction of degradation model is detailed. Section 3 develops quantities of interest which are useful in the reliability analysis, namely the distribution of the failure time and the distribution of the remaining lifetime of the unit. Section 4 compares approaches for identifying properties of the model in terms of statistical inference. Finally, Section 5 presents a numerical example illustrating the proposed methodology for model validation.

2. Degradation processes and Random field variability

2.1. Standard time variant Gamma process modeling

The standard Gamma process (GP) is an appropriate mathematical model for modeling the degradation evolution in structural engineering, such as corrosion and cracks of materials, which are the common causes of structural failure. The deterioration is supposed to take place gradually over time in a sequence of tiny increments. Consider $\alpha(\cdot)$ to be a non-decreasing, right-continuous, real-valued function for $t \geqslant 0$ and vanishing at $t = 0$.

Definition 2.1. A stochastic process $(X_t)_{t\geqslant 0}$ is said to be a GP with shape function $\alpha(t)$ and identical scale parameter $\beta > 0$ if the process satisfies the following properties:

- $X_0 = 0$ with probability one,
- The increment $X_{t+s}-X_t$ has a Gamma distribution $Ga(\alpha(t+s)-\alpha(t), \beta)$,
- X_t has independent positive increments,

where the Gamma distribution $Ga(\alpha, \beta)$ is defined by the density function:

$$f_{Ga}(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, \text{ for each } x > 0, \tag{1}$$

and Γ is the classical Gamma function. The process X_t is said to be stationary if $\alpha(\cdot)$ is a linear function and X_t is non-stationary if $\alpha(\cdot)$ is a non-linear function. The mean and variance of X_t are $\mathbb{E}[X_t] = \frac{\alpha(t)}{\beta}$, $var[X_t] = \frac{\alpha(t)}{\beta^2}$, respectively. The process X_t satisfies the scaling property,

$$\gamma X_t = Ga(\alpha, \beta/\gamma), \text{ for each } \gamma > 0, \tag{2}$$

and its logarithm $\log(X_t)$ has the following two first moments:

$$\mathbb{E}[\log(X_t)] = \psi(\alpha(t)) - \log(\beta), \tag{3}$$

$$var[\log(X_t)] = \psi_1(\alpha(t)), \tag{4}$$

where the function ψ is digamma function which is defined as the logarithmic derivative of Γ, and ψ_1 is the trigamma function defined as the derivative of ψ.

2.2. Gaussian spatial random field modeling

Spatial variability of material properties is classically modeled by a second-order stationary random field (RF) given by a non-linear transformation $\mathscr{T}(\cdot)$ of a Gaussian random field (GRF) $\mathscr{T}(Y)$ [8,31]. For example, in the transport equation, the diffusivity coefficient is modeled by a log-normal RF [1,2,19], where its distribution is obtained as a limit of physical positive quantities.

We consider $Y(z, \omega)$ to be a spatial GRF in the set $D \times \Omega$, where D is a set in $\mathbb{R}^d$ with $d = 1, 2, 3$ and Ω is an abstract set of events. The GRF Y is assumed to be homogenous and then totally defined by its mean $\mu \in \mathbb{R}$ and its covariance function $cov(r)$ [7,36]. The $cov(r)$ function models the correlation between two spatial random variables on any two points separated by the distance r.

In order to simulate Y on $\{z_0, z_1, ..., z_N\} \subset D$ a set of equidistant points, we choose the circulant embedding matrix approach [14] (also named by DSM discrete spectral method in [11]). This method is a very versatile approach for generating GRF, the discretized RF has the same spatial correlation on the grid points. In [30] the authors develop a continuous spectral method to simulate Y where the spectral density is discretized on a uniform grid, then a Discrete Fourier Transform (DFT) is used to generate an approximation of Y. The proposed simulated GRF is asymptotically Gaussian where its correlation structure is an approximation of the target correlation and its accuracy is strongly related to the regularity of Y (see [22]). Another widely used approach to