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Influence of boundary conditions and load eccentricity on strength of cold-formed lipped channel columns

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ABSTRACT

The ultimate buckling strength of axially loaded cold-formed lipped channel columns is studied. Variations of boundary conditions and eccentricities of the applied load are considered. The strength is calculated by a geometrically and materially nonlinear FEM analysis. Numerical study is carried out for two columns previously analysed under fixed boundary conditions and centroidal load. Geometrical imperfections in the shapes of eigenmodes of the corresponding linearized buckling problem are assumed. For comparing the imperfections their sizes are set to a uniform level of an energy measure. Particularly the worst eigenmode imperfection is determined for each studied design case. By the corresponding lowest collapse loads the effects of varying boundary conditions and load eccentricities on the ultimate buckling strength of the columns are shown.

1. Introduction

Generally, the behaviour and ultimate buckling strength of thin-walled structures and structural elements is significantly influenced by imperfections. Particularly, the geometrical imperfections considered as deviations from the initially perfect mid-surface geometry are of main concern. Both their shapes and sizes are of importance. Because of the lack of representative number of measured data, theoretical imperfection shapes are employed in computational assessment of the ultimate buckling strength. Usually, the eigenmodes of elastic buckling problem as well as periodic shapes described by trigonometric functions or combinations of thereof are applied.

For comparison of imperfections aiming at finding the most unfavourable one, a measure of their sizes has to be adopted. Commonly utilised amplitude is a local measure not showing the level of overall deformations. This point is partly taken into account by differentiating few basic modes often relating their amplitudes to the standards of execution tolerances or measurements statistics. In cold-formed steel members mostly three basic modes: local, distortional and global are considered. Nevertheless the problem of comparing slightly and heavily deformed shapes remains, e.g., modes of few as well as of numerous local half waves of equal amplitudes are assumed acceptable for comparison. Another problem with the local measure arises when combinations of the basic modes are sought. One may ask what should be the amplitude of the combined mode when individual components possess different amplitudes. Moreover, since amplitude can not be used as a unique measure of imperfections it may obstruct advancing to probabilistic approaches.

Selection of influential imperfection shapes by which the worst imperfection can be determined is a challenging task. The eigenmodes of the related eigenvalue problem, known as incipient shapes of the nonlinear solutions bifurcating from the idealised perfect state at the corresponding buckling loads, are naturally employed. Distinct distribution of buckling loads suggests that the first or second eigenmode may be decisive. When higher buckling loads are situated near the critical buckling load, the group of imperfection modes of potential importance can increase significantly. Nevertheless, since the eigenvalue problem relates to small nonlinear solutions such observations should be regarded as indications only. One can recall the well known possibility of occurrences of smooth, bifurcational or snap through changes of nonlinear solutions. Despite of this, thoughtful utilisation of indications derived from the eigenvalue solutions may prove helpful.

Rapid progress in the use of geometrically and materially nonlinear FEM simulations with imperfections (GMNIA) for the design of structures, see e.g. [1–4], escalated the demand for guidance on inclusion of imperfections. The informative Annex C “Finite Element Methods of Analysis (FEM)” of EN 1993-1-5: 2006 is known as the first attempt to codify implementation of imperfections in the nonlinear FEM analysis for design purposes. The geometrical imperfections are based on eigenmodes scaled to 80% of fabrication tolerances. Choosing any one as

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a leading imperfection, possible accompanying modes should be reduced to 70% of the former. For achieving the worst case, one imperfection a time should be tried as leading. Likewise accompanying modes (one or several) are included.

The current evolution of Structural Eurocodes (CEN/TC 250) aims at the development of standardized design guidance for computational FEM modelling of structures, which should provide more economical design without reducing target levels of safety. It should be based on consistent allowances for structural imperfections and specification of geometrical execution tolerances with regard to buckling strength of columns, frames, plated structures and shells. A contribution to the discussion on achievement of this goal was published in our previous paper [5]. The suggested approach is based on an overall — energy measure of geometrical imperfections. It was applied on cold-formed lipped channel columns of three cross-sectional geometries and several column lengths. In GMNIA computational implementation of FEM fixed boundary conditions and centroidal axial load were adopted.

The concept of energy measure (*EM*) of geometrical imperfections was published in [6]. The measure derives from the elastic strain energy hypothetically required to deform a perfect structural element into the considered imperfect shape [7]. Defining *EM* by the square root of the energy means its equivalence to the norm in the functional space of variational solutions of elastic eigenvalue buckling problems. The norm is related to a scalar product by which the eigenmodes form an orthogonal system which is complete in that space. The combinations of eigenmodes simply follow the Pythagoras triangle rule. A minimization of the potential energy of an abstract buckling problem (described in terms of operators in that space) perturbed by small initial imperfections of a constant energy measure (norm) was performed in [6]. The minimization was regular for loads below the critical buckling load providing overall and initial deflections in the shapes of buckling modes as stationary points of this constrained minimization problem. That point of view corroborates the use of eigenmodes as characteristic imperfection shapes, however under the energy measure normalization.

The energy measure of eigenmode imperfections is another quantity derived from the solution of eigenvalue problem which can facilitate the choice of influential eigenmodes [5,8]. Normalizing the eigenmodes by the unit maximum displacement component (co-ordinate shift from the perfect state) realized in NASTRAN, ABAQUS, ANSYS or COSMOS/M, the corresponding *EM* value of an eigenmode reflects the amount of imaginary strain energy stored in that mode. The lesser is the energy necessary for reaching the unit magnitude of an eigenmode, the greater is buckling deformability of that mode. Normalizing the eigenmodes by the energy measure, the greater buckling deformability of a mode is indicated by its higher amplitude (the maximum of co-ordinate shifts). Adopting normalization of imperfections by the energy measure *EM* [8], the amplitudes of normalized eigenmodes are used as a comparative parameter of their buckling deformability.

For description of the approach suggested in [8] it is proper to consider equalities defining the energy measure of eigenmodes in FEM approximation:

$$EM(\varphi_i) = \left(\frac{1}{2}\varphi_i^T K_e \varphi_i\right)^{1/2} = \left(\lambda_i \frac{1}{2}\varphi_i^T K_g \varphi_i\right)^{1/2}. \tag{1}$$

K_e and K_g are the elastic stiffness and geometric matrices, respectively. φ_i denotes the *i*-th eigenmode and λ_i represents its buckling load P_{cri}. Both the norm and the scalar product are determined by the stiffness matrix K_e. The right-hand side equality of Eq. (1) derives from the *i*-th solution of the eigenvalue problem showing that the strain energy of φ_i is equal to one half the work of external forces. Thereby, the quadratic form generated by the geometric matrix K_g represents the axial shortening related to the *i*-th eigenmode, i.e. characterizes the axial deformability of that mode. One would naturally expect that the axial deformability decreases with the modes order number, i.e. the higher the buckling load, the greater the axial stiffness of the corresponding mode. This exactly happens when normalizing the eigenmodes by the energy measure, as shows Eq. (1) for constant *EM* [8]. When normalizing the eigenmodes by amplitude this feature is not observed. The buckling loads are used as a discrete parameter of axial stiffness of the corresponding modes.

The procedure facilitating the choice of influential modes starts with normalization of geometrical imperfections by the energy measure [8]. A basic level of *EM* normalization is adjusted by reaching the unit maximum amplitude (determined in terms of co-ordinate translations) within the considered set of eigenmodes. A graphical presentation showing buckling loads along with the amplitudes of the normalized eigenmodes is required. The heavily deformed modes possessing small amplitudes generally show a branch of values decreasing with their order number Fig. 3 [5] and Figs. 3–5, etc. Eigenmodes of great axial stiffness (corresponding to high buckling loads) and little buckling deformability (of lower amplitudes) were observed as not influential [5,8]. Modes of lower or moderate axial stiffness (close and not too far above the critical buckling load) as well as those of significant buckling deformability (of upper cross-sectional amplitudes) are candidates for the most influential eigenmode imperfection to be checked by collapse loads calculations. The power of the axial stiffness parameter also depends on the *EM* level at which the eigenmode imperfections are compared. Decrease of the level increases significance of the axial deformability parameter. For example at minute amplitudes ($<$ 0.01 mm) the first eigenmode shape was the decisive imperfection of a lipped channel column having thickness of 1.5 mm [8]. Increase of the *EM* level enhances significance of the buckling deformability parameter as showed failure load computations at moderate and tolerances limited *EM* levels, i.e. higher eigenmodes may become influential, cf. Fig. 10 and 17 [5].

In earlier GMNIA studies on post-buckling of plates unexpected strength values were obtained using eigenmode imperfections of equal amplitudes. Dow and Smith [9] dealing with rectangular plates of aspect ratio 4 subjected to longitudinal compression found that the strength of plates with initial deflections of 4, 5 and 6 half-waves decreased with the number of half-waves. Similar results were already reported in [10]. In [11] the compressive strength of rectangular plates of aspect ratio 3 and three slenderness ratios was studied considering theoretical imperfections and measurements of ships plating distortions. Using eigenmode imperfections with amplitudes within the range derived from measurements consistent results with the aforementioned were obtained. However, the strength values for imperfections of 3–5 half-waves were significantly below the cloud of plates capacities obtained for measured imperfections. When adopting normalization by the energy measure within the *EM* ranges of measurements, the scatters of strength values corresponding to theoretical and measured imperfections reasonably correlated.

The present paper extends the investigation of ultimate buckling strength of two cold-formed lipped channel columns carried out in [5] by releasing fixed boundary conditions. Rotations of the loaded end cross-section about the minor and major axis as well as about the longitudinal axis are allowed. Warping of end-cross sections is prevented. Further, eccentric load applications in the points of the major or minor axis as well as a combined eccentricity are assumed. For modelling the outlined boundary and load conditions, stiff flat platens are attached at the end cross-sections of the channel. The main goal is to show performance of the procedure for selection of the most influential eigenmodes in new types of design cases. Focus is placed on finding the worst eigenmode imperfection under moderate initial distortion level. Combination of eigenmodes or adjustment of imperfections sizes by execution tolerances is not addressed here.

The energy measure is also used for comparison of ultimate buckling strength values at different boundary conditions and load eccentricities. Assuming that the characteristics of initial deformations are derived from built in members before operation, the collapse loads of an individual column are calculated for imperfections having the same