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# A nonlinear second-order filtering strategy for state estimation of uncertain systems

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ABSTRACT

In this paper, a new strategy referred to as the nonlinear second-order (NSO) filter is presented and used for estimation of linear and nonlinear systems in the presence of uncertainties. Similar to the popular Kalman filter estimation strategy, the proposed strategy is model-based and formulated as a predictor-corrector. The NSO filter is based on variable structure theory that utilizes a switching term and gain that ensures some level of estimation stability. It offers improvements in terms of robustness to modeling uncertainties and errors. The proof of stability is derived based on Lyapunov that demonstrates convergence of estimates towards the true state values. The proposed filtering strategy is based on a second-order Markov process that utilizes information from the current and past two time steps. An experimental system was setup and characterized in order to demonstrate the proposed filtering strategy's performance. The strategy was compared with the popular Kalman filter (and its nonlinear form) and the smooth variable structure filter (SVSF). Experimental results demonstrate that the proposed nonlinear second-order filter provides improvements in terms of state estimation accuracy and robustness to modeling uncertainties and external disturbances.



## 1. Introduction

Estimation is the process of extracting true state and parameter values from systems in the presence of noisy measurements, modeling uncertainties, and unwanted disturbances. This task aims to provide optimal estimates in terms of minimal estimation error, which is defined as the difference between the estimated and actual state values. Inherent to the estimation process is system and measurement noise, external disturbances, and uncertainties–all of which can be caused by sensors, instruments, or the environment. In order to overcome these issues, model-based estimation and filtering strategies are utilized to monitor and control engineering systems. In model-based methods, a probability density function (PDF) is calculated recursively, and is based on the state estimates. Information on the state mean and state covariance is contained within the PDF, and can be used to provide state estimates. Model-based strategies are recursive, and consist of two stages: predict and update. In the first stage, the system model is used to estimate (or predict) the state values at the next time step. The update stage, as the name suggests, refines the predicted state estimates based on system measurements. The most popular model-based method used for linear estimation problems is the well-known Kalman filter (KF) [1]. The KF assumes that the estimation problem is linear, the system is known, and the noise is zero-mean and Gaussian distributed. For general nonlinear and non-Gaussian systems, several strategies have been proposed: linearization (e.g., the extended Kalman filter or EKF [2,3]), and PDF approximation (e.g., the unscented Kalman filter or UKF [3], the cubature Kalman filter or CKF [4]). It has been demonstrated that the CKF is merely a special case of the UKF [4]. Due to improvements in computing power and reductions in cost, particle filters (PFs) have grown in popularity [5]. Similar to the UKF, the PF uses a large set of weighted particles that approximate the state PDF [3,6].

One of the main issues with the KF is that the estimation performance may degrade in the presence of modeling and parameter uncertainties. To overcome this issue, robust state estimation techniques are implemented, such as minimax estimators, worst-case, or set-membership state estimators [7,8]. From a statistical standpoint, the minimax estimators deal with uncertainties that are uniformly distributed within given bounds. In the case of ellipsoidal bounding sets, these estimators coincide with the KF for linear systems. Interestingly, there also exists minimax estimators where the uncertainty is mathematically expressed using entropy-like indexes [9]. Based on propagation of uncertainties, a family of

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**Nomenclature**

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|---|---|
| **A** | Linear state matrix |
| $\mathbf{A_E}$ | Piston area |
| **B** | Linear control matrix |
| $\mathbf{B_E}$ | Load friction |
| $\mathbf{D_P}$ | Pump displacement |
| **H** | Linear measurement matrix |
| **K** | Filter's gain |
| **L** | Leakage coefficient |
| **M** | Load mass |
| **P** | State error covariance matrix |
| **Q** | Process noise covariance matrix |
| $\mathbf{Q_e}$ | Leakage flow rate |
| $\mathbf{Q_{L0}}$ | Flow rate offset |
| **R** | Measurement noise covariance matrix |
| **S** | Vector of sliding variables |
| **T** | Sampling rate |
| $\mathbf{V_0}$ | Initial cylinder volume |
| $\mathbf{a_1,a_2,a_3}$ | Friction coefficients |
| **e** | Estimation error |
| **f** | Nonlinear state model |
| **k** | Sample time |
| **s** | Sliding mode variable |
| **u** | Control variable |
| **v** | Measurement noise |
| **w** | Process noise |
| **x** | State vector |
| **z** | Measurement vector |
| $\boldsymbol{\beta_e}$ | Effective bulk modulus |
| $\boldsymbol{\gamma}$ | Convergence rate |
| $\varepsilon$ | Upper bound |
| $\boldsymbol{\omega_P}$ | Motor rotational velocity |
| $\psi$ | Smoothing boundary layer |
| $\hat{\square}$ | Estimated quantity |
| $\square^{+}$ | Pseudo-inverse operator |

robust Kalman filter may be derived [10]. Other robust strategies include the so-called robust KF [11,12] and the H∞ filter [13]. The robust KF was used for systems with bounded modeling uncertainties such that an upper bound of the mean square estimation error (MSE) is minimized at each step [11]. Considerable research has been performed on the design of robust state estimation methods for dynamic systems with bounded uncertainties, such as minimax estimators [14], worst-case [7,15], or set-membership state estimators [8]. Zames [13] created the H∞ method by removing the necessity of a perfect model or complete knowledge of the input statistics. The H∞ theory was designed by tracking the magnitude of the 'energy' of a signal for the worst possible scenario in terms of noise levels and modeling uncertainties.

In 2007, an initial form of the smooth variable structure filter (SVSF) was introduced based on variable structure theory introduced in the 1970s [16,17]. Similar to the KF method, the SVSF [17] is a predictor-corrector strategy. However, the SVSF formulation is unique since the gain is derived based on a discontinuous corrective gain. This gain bounds state estimates to within a region of the true state trajectory, improving stability of estimates and robustness to external disturbances [17]. The discontinuous corrective action provided by the SVSF gain has demonstrated robustness to bounded modeling uncertainties [18,19]. A smoothing term (e.g., saturation function) is used to suppress or smooth chatter caused by the SVSF gain [20]. However, the robustness of the method comes at a trade-off; the SVSF introduced in 2007 is a sub-optimal filter [21,22]. Gadsden extended the SVSF by deriving a state error covariance term for it, and using the term to obtain an optimal smoothing boundary layer [18,19]. Results demonstrate improved state estimation while maintaining robustness to modeling uncertainties and disturbances [18,19,23]. Afshari et al. have researched on the design and application of hydraulic and pneumatic actuation systems. They implemented a number of techniques to analyze the dynamic behavior of such systems [24,25]. Moreover, Afshari et al. investigated the performance of popular robust estimation methods with applications to fault detection and diagnosis [26–29], maneuver vehicle tracking [30–32], and energy management systems [33,34].

This paper is motivated by state estimation problems for systems with modeling uncertainties or errors, such as in fault operating conditions. Since a higher-order version of the SVSF is derived, it is expected that the proposed method will yield a more accurate solution to the estimation problem in terms of state error. However, the higher-order accuracy comes at a trade-off with computational complexity and time. Since computers are being extremely fast and relatively cheap, the issue of computational power requirements is less important than a decade ago. During system faults, the mathematical model of the system used by the filter deviates from the true model (e.g., normal conditions). In most cases it is extremely difficult (or impossible) to identify all of the possible operating and fault conditions. The proposed NSO filter, described in Section 2, is able to overcome this issue by generating state estimates for systems subjected to 'soft' fault conditions. The stability of the proposed filter is proven mathematically. Different measurement cases (full and reduced) for the proposed filter are described in Sections 3 and 4, respectively. An experimental setup was used to verify and compare the proposed NSO filter with the popular KF and the EKF. As described in Section 5, two cases were studied: linear system with only one measured state, and nonlinear system with full measurements. The paper is concluded in Section 6.

## 2. NSO filtering strategy

The proposed NSO filter is based on the SVSF, whereby a second-order formulation of the gain is implemented [17]. The strategy can be formulated to work with linear and nonlinear systems. However, for nonlinear systems without full measurements, the nonlinearities need to be linearized or approximated. A technique is presented in [17] to obtain the gain for unmeasurable states of a nonlinear system without the need for linearization. The proposed filter utilizes a prediction and update stage (described in this section). To formulate the NSO filter, consider a nonlinear system represented by a discrete state model as follows:

$$\mathbf{x}_{k+1} = \mathbf{F}(\mathbf{x}_k, \mathbf{u}_k, \mathbf{w}_k), \tag{1}$$

where $\mathbf{F} : \mathbb{R}^{2n+p} \to \mathbb{R}^n$ is the nonlinear state model, $\mathbf{x}_k \in \mathbb{R}^{n\times 1}$ is the state vector, $\mathbf{u}_k \in \mathbb{R}^{p\times 1}$ is the control vector, and $\mathbf{w}_k \in \mathbb{R}^{n\times 1}$ is the process noise (modeling uncertainties) vector. The measurement model is assumed to be linear or at least piece-wise linear such that:

$$\mathbf{z}_k = \mathbf{H}\mathbf{x}_k + \mathbf{v}_k, \tag{2}$$

where $\mathbf{z}_k \in \mathbb{R}^{m\times 1}$ is the measurement vector, $\mathbf{v}_k \in \mathbb{R}^{m\times 1}$ is the measurement noise, and $\mathbf{H} \in \mathbb{R}^{m\times n}$ is the measurement matrix.

*Assumption 1:* The control vector $\mathbf{u}_k$ is assumed known and norm-bounded. Moreover, vectors $\mathbf{w}_k$ and $\mathbf{v}_k$ are assumed to be unknown but norm-bounded, and with a zero mean.

*Assumption 2:* It is assumed that the system with Eqs. (1) and (2) is smooth and with continuous partial derivatives.

Based on these assumptions, consider the following steps for the NSO filter.