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The noisy voter model under the influence of contrarians

Nagi Khalil *, Raúl Toral

IFISC (CSIC-UIB), Instituto de Física Interdisciplinar y Sistemas Complejos, Campus Universitat de les Illes Balears, E-07122 Palma de Mallorca, Spain

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ABSTRACT

The influence of contrarians on the noisy voter model is studied at the mean-field level. The noisy voter model is a variant of the voter model where agents can adopt two opinions, optimistic or pessimistic, and can change them by means of an imitation (herding) and an intrinsic (noise) mechanisms. An ensemble of noisy voters undergoes a finite-size phase transition, upon increasing the relative importance of the noise to the herding, form a bimodal phase where most of the agents share the same opinion to a unimodal phase where almost the same fraction of agent are in opposite states. By the inclusion of contrarians we allow for some voters to adopt the opposite opinion of other agents (anti-herding). We first consider the case of only contrarians and show that the only possible steady state is the unimodal one. More generally, when voters and contrarians are present, we show that the bimodal-unimodal transition of the noisy voter model prevails only if the number of contrarians in the system is smaller than four, and their characteristic rates are small enough. For the number of contrarians bigger or equal to four, the voters and the contrarians can be seen only in the unimodal phase. Moreover, if the number of voters and contrarians, as well as the noise and herding rates, are of the same order, then the probability functions of the steady state are very well approximated by the Gaussian distribution.

1. Introduction

Nowadays it is quite common to model the dynamics of opinion as a complex system in terms of agent-based models. In those models "agents" or "units" can hold different opinions that evolve under dynamical rules that include stochastic effects. In this framework, the global behaviour of the system can be directly linked to the microscopic mechanisms acting at the level of one or a few agents. The voter model (VM) [1–4] and the majority rule model (MR) by Galam [5–8] are paradigmatic examples of agent-based models where each agent can be in one of two possible opinion states and the dynamics is driven by an imitation process. For the VM a randomly chosen agent blindly copies the state of a neighbour, again randomly chosen, while in the MR a complete group of randomly chosen agents adopt the opinion of the local majority. For finite systems, both models describe an evolution towards a consensus state where all agents share the same opinion [3,9].

In the real world, however, perfect consensus is an exception and coexistence of opinions is a more likely stable scenario. Both the VM and the MR have been modified in different ways in order to account for this more realistic situation. Among many possibilities, it has been shown that the inclusion of inflexible agents (also known as zealots) or that of contrarians prevents the system from reaching a perfect consensus state, allowing coexistence to prevail. Zealots are agents that never change their opinion, their influence depending both on their number and the detailed structure of the network of

* Corresponding author.
E-mail address: nagi@ifisc.uib-csic.es (N. Khalil).

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interactions [10–18]. Contrarians are agents that, contrarily to the imitation rules described above, tend to copy the opposite opinion of a neighbour or to adopt the opinion held by the minority of the group. Their influence on the MR was first studied by Galam [19]. He showed that the system can reach two steady states, depending on the concentration of contrarians: if it is small enough, an ordered phase is reached with a majority (but not all) of agents holding one of the two opinions; while for the concentration above a critical value, a disordered phase is reached with the same fraction of agents with different opinions (no majority). See Refs. [20–35], amongst others, where contrarians have also been considered in other contexts.

Contrarian behaviour has been also studied within the VM, in the mean-field approximation with all agents being neighbours, in Refs. [36,37]. In the two-role model (TRM) of [36,37] the agents can choose, at each decision step, between behaving as a "voter" and then copy the opinion of a neighbour, or behaving as a "contrarian" and adopt the opposite opinion of a neighbour, with given probabilities $1-p$ and p, respectively. Observe that the situation is different from the original model by Galam [19] where agents have fixed roles and the label of "voter" or "contrarian" of a single agent remains unchanged during the dynamical evolution. For a system of N agents, three phases can be observed: the bimodal phase if $p<1/(N+1)$, the plain phase[1] for $p=1/(N+1)$, and the unimodal phase for $p>1/(N+1)$. In the bimodal phase, the system keeps most of the time close to the consensus states, where the number of agents holding one particular opinion is much larger than the number holding the opposite one, but the dominant opinion can change with time. Hence, the probability distribution $P(n)$ of the number of agents n holding a particular opinion has maxima at $n=0$ and $n=N$ in the steady state. In the unimodal phase the number of agents in each of the two possible states fluctuates around equal numbers and the distribution $P(n)$ presents a single maximum at $n=N/2$. In the plain phase, $P(n)$ is uniform for all $n\in[0,N]$.

The situation of the TRM resembles that of the Kirman or noisy voter model [38–41]. In that model there are two mechanisms that make an agent change her binary state: the herding or copying mechanism, as in the VM, and the intrinsic noise allowing agents to change states regardless of the state of the remaining agents. By increasing the relative importance of the noise with respect to the herding, the system undergoes a finite-size phase transition from a bimodal to a unimodal phase, similar to the one observed in the TRM with respect to the probability p.

In this work we study the effect of contrarian agents on the noisy voter model. As in Galam's model, agents are modelled with fixed roles that they keep at all times. The main objective is to unveil the effects of the different mechanisms on the global behaviour of the system, specially on the different phases the system may exhibit, as well as to clarify the similarities and differences between the noisy voter model and the two-role model.

The paper is organized as follows. In the next section we introduce the model and explore some limits, particularly the case of the TRM. Section 3 contains the main results of the system: exact and approximate theoretical expressions are compared against numerical results for simple as well as general cases. Finally, Section 4 is devoted to the conclusions.

2. Model

The system is made of $N=N_v+N_c$ agents, where the suffixes v and c stand for the voters and the contrarians, respectively. Each agent can hold any of two possible opinion states. We do not have any particular interpretation in mind, and we will denote the two states generically as "up" and "down". They could account for "optimistic" and "pessimistic" states; the "buy" and "sell" states of brokers in the stock market, or whatever other interpretation. We assume all agents inside each subgroup to be equivalent, hence the state of the system is fully specify by the set $\{n_v,n_c\}$ of the numbers of up voters n_v and up contrarians n_c, with $n\equiv n_v+n_c$ being the total number of up agents. Both voters and contrarians can change their state randomly with certain rates, increasing and decreasing the number n_v and n_c. The dynamics implements a Markov chain with the rates $\pi^{\pm}_{v,c}$ for the allowed transitions that differ for voter and contrarian agents.

1. Voter agents. A voter agent can change her state either randomly at a rate a_v or by a copying mechanism that occurs at a rate proportional (with a proportionality constant h_v) to the fraction of agents holding the opposite state to the given agent. Hence, the number of voters in the up state, n_v, can decrease or increase by one with the following rates:
   - $n_v\to n_v+1$. It is needed that any of the N_v-n_v voter agents in the down state switches to the up state, either with rate a_v or with rate $h_v\frac{n}{N}$, as $\frac{n}{N}$ is the fraction of agents in the opposite (up) state. The total rate of this process is
   $$\pi^+_v(n_v,n_c)=\left(a_v+h_v\frac{n}{N}\right)(N_v-n_v), \tag{1}$$
   - $n_v\to n_v-1$. It is needed that any of the n_v agents in the up state switches to the down state, either with rate a_v or with rate $h_v\frac{N-n}{N}$, as $\frac{N-n}{N}$ is the fraction of voters in the opposite (down) state. The total rate of this process is
   $$\pi^-_v(n_v,n_c)=\left(a_v+h_v\frac{N-n}{N}\right)n_v, \tag{2}$$
2. Contrarian agents. A contrarian agent still changes state at a constant rate a_c but interacts with the rest of agents by changing her state with a rate proportional (with a proportionality constant h_c) to the fraction of agents that hold the same state than the given agent. Hence, the number of contrarians in the up state, n_c, can decrease or increase by one with the following rates:

[1] Please, note that in most of the literature the plain phase is considered as a boundary or a border case and not a phase in itself.