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Fractality of evolving self-similar networks

Jialing Yao, Bingbin Sun, lifeng Xi *

Department of Mathematics, Ningbo University, Ningbo, 315211, PR China

HIGHLIGHTS

- A family of evolving self-similar networks is constructed.
- Our self-similar model is rigorously deterministic.
- The fractality exists in our self-similar model.

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ABSTRACT

Self-similarity plays an important role in the study of fractal networks. In this paper, we construct a class of self-similar evolving networks by replacing one node with an initial graph. Our substitution rule is based on the directed graph and then the corresponding networks are deterministic. Moreover, we explore the fractality of our evolving self-similar networks.

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1. Introduction

In the past 20 years, the research on complex networks has attracted wide attention. For example, Watts and Strogatz [1] put forward a small world model which depicts the characteristics of large clusters and short average path distance in real networks, Barabási and Albert [2] proposed scale-free network model which has growth and preferred connections, Newman [3–5] studied the structure and function of complex networks, and Milo et al. [6] investigated simple building blocks of complex networks. For more information about complex networks, please refer to [7–14].

Self-similarity of irregular geometry was introduced by Mandelbrot [15], the founder of fractal geometry. Using box counting method of fractal geometry, Song et al. [16–19] introduced a characteristic named fractality for complex networks. Kim et al. [20] studied the fractality and self-similarity in scale-free networks, Gallos et al. [21–23] investigated functional brain networks which are fractal networks, Zhang et al. [24] researched Vicsek fractal networks. Please also refer to [25–27] for applications of self-similarity and fractality to complex networks in biology.

Li et al. [28] constructed the evolving self-similar networks in terms of substitution rule which replace edges of different colors with different initial graphs and obtained the free-scale effect of their networks.

What will happen if we turn a node into an initial graph? In fact, Song et al. [18] studied this growth mechanism. We consider an initial graph $G_1 = G = (V, E)$ with $\#V = m$. By induction, assume that the graph G_{t-1} has been constructed. Let nodes x and y be neighbors in G_{t-1}, at the next time t, they are replaced by $G^{(x)}$ and $G^{(y)}$ respectively which are copies of the initial graph G. We delete the edge from x to y in G_{t-1} and add an edge (of G_t) from one node in $G^{(x)}$ to another node in $G^{(y)}$, where these two nodes are chosen randomly. It is noteworthy that the above growth process is not rigorously deterministic if the initial pattern G is not symmetric.

* Corresponding author.
E-mail addresses: 1711071027@nbu.edu.cn (J. Yao), 1711071020@nbu.edu.cn (B. Sun), xilifengningbo@yahoo.com (l. Xi).

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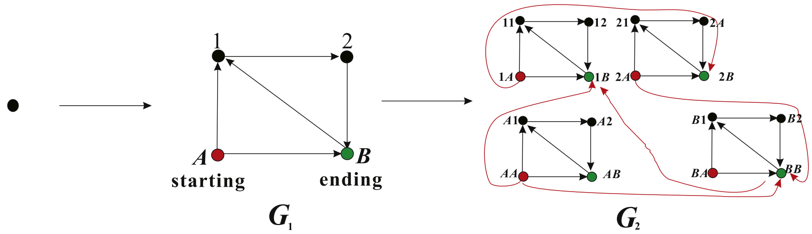

Fig. 1. An example with an unsymmetric initial pattern.

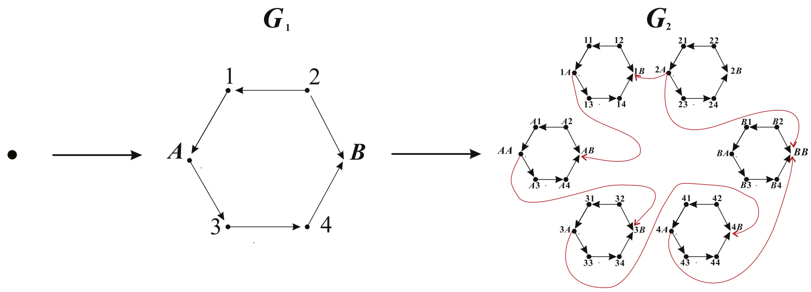

Fig. 2. Example 1.

In this paper, we use the directed graphs to revisit the above growth mechanism, inspired by the rigorously deterministic idea in [29]. We take a directed graph G as the initial graph. Fix two nodes in G, we assign one (says A) as starting node and another (says B) as ending node. Now, the substitution rule is that for an edge from node x to node y in G_{t-1} we delete the edge, replace x, y with the initial graph $G^{(x)}$ and $G^{(y)}$ respectively and add a directed edge from the starting node of $G^{(x)}$ to the ending node of $G^{(y)}$. We obtain a family of evolving networks $\{G_t = (V_t, E_t)\}_t$ by induction. Denote by $\tilde{G}_t$ the modified undirected graph with respect to G_t. Finally, we get a family of undirected evolving networks $\{\tilde{G}_t = (V_t, \tilde{E}_t)\}_t$ (see Fig. 1).

We recall some notions of fractal network by Song et al. [16,17]. An l-box is a subset of node set V such that the shortest distance between any two nodes in the subset is less than l, and fractal dimension α is given by $\frac{\#V}{N(l)} \sim l^{\alpha}$, where $N(l)$ is the smallest number of l-boxes needed to cover the network.

We need to define an index for a undirected path in $\tilde{G}$. Given a undirected path $\gamma = \tilde{e}_1 \cdots \tilde{e}_k$ passing nodes $v_0, \ldots, v_k$, where $e_i \in E$ for all i and $\tilde{e}_i$ connects v_{i-1} with v_i. If the e_i is from v_{i-1} to v_i, we say the edge e_i has positive direction, otherwise say e_i has negative direction. Let

$$\alpha_i = \begin{cases} 1 & \text{if } e_i \text{ and } e_{i+1} \text{ have the same direction,} \\ 0 & \text{otherwise.} \end{cases}$$

Adding two directed edges to the path, one is from v_0 to a dummy node, another is from a dummy node to v_k, then we can define α_0 and α_k as above. Denote

$$\text{index}(\gamma) = \sum_{i=0}^{k} \alpha_i.$$

For x, y in $\tilde{G}$, let $n(x, y)$ be the smallest index of all paths from x to y in $\tilde{G}$, and

$$n = \min\left(n(A, B), n(B, A)\right).$$

The following result of this paper implies that the fractality exists and the fractal dimension depends heavily on the choice of the starting node and the ending node.

Theorem 1. *Suppose $n > 1$. Then self-similar networks $\{\tilde{G}_t\}_t$ satisfy the fractality, i.e.,*

$$\frac{\#V_t}{N_t(l)} \sim l^{\frac{\log m}{\log n}},$$

where $N_t(l)$ is the smallest number of l-boxes needed to cover V_t.

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