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A generalized framework for chance-constrained optimal power flow

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A B S T R A C T

Deregulated energy markets, demand forecasting, and the continuously increasing share of renewable energy sources call – among others – for a structured consideration of uncertainties in optimal power flow problems. The main challenge is to guarantee power balance while maintaining economic and secure operation. In the presence of Gaussian uncertainties affine feedback policies are known to be viable options for this task. The present paper advocates a general framework for chance-constrained OPF problems in terms of continuous random variables. It is shown that, irrespective of the type of distribution, the random-variable minimizers lead to affine feedback policies. Introducing a three-step methodology that exploits polynomial chaos expansion, the present paper provides a constructive approach to chance-constrained optimal power flow problems that does not assume a specific distribution, e.g. Gaussian, for the uncertainties. We illustrate our findings by means of a tutorial example and a 300-bus test case.

1. Introduction

The continuing increase in electricity generation from renewable energy sources and liberalized energy markets pose challenges to the operation of power systems [1]; i.e., the importance of uncertainties is on the rise. Uncertainty leads to and/or increases fluctuating reserve capacities, and varying line power flows across the network, among others. The structured consideration of uncertainties is thus paramount in order to ensure the economic and secure operation of power systems in the presence of fluctuating feed-ins and/or uncertain demands.

Optimal power flow (OPF) is a standard tool for operational planning and/or system analysis of power systems. The objective is to minimize operational costs whilst respecting generation limits, line flow limits, and the power flow equations. Assuming no uncertainties are present the solution approaches to this optimization problem are numerous, see for example references listed in [2]. In the presence of stochastic uncertainties the OPF problem must be reformulated, ensuring

(i) that technical limitations (inequality constraints) are met with a specified probability, and
(ii) that the power flow equations (equality constraints) are satisfied for all possible realizations of the uncertainties, i.e. power system stability is achieved.

Regarding issue (i), chance-constrained optimal power flow (CC-OPF) is a formulation that allows inequality constraint violations with the probability of constraint violation as a user-specified parameter. Individual chance constraints admit deterministic, distributionally robust convex reformulations of the CC-OPF problem [3]. For Gaussian uncertainties these reformulations are exact [3–5]. Alternatively, it is possible to solve the individually chance-constrained optimization problem by means of multidimensional integration [6,7]. Scenario-based methods – often applied to multi-stage problems [8–10] – are an alternative to chance-constrained approaches; the chance constraints are replaced by sufficiently many deterministic constraints leading to large but purely deterministic problems [11].

Regarding issue (ii), the power flow equations are physical constraints that hold despite fluctuations. This requires feedback control. In particular, automatic generation control (AGC) balances mismatches between load and generation, given sufficient reserves can be activated. Affine policies have been shown to yield power references that satisfy DC power flow in the presence of (multivariate) Gaussian uncertainties [3–5,12] (assuming ideal primary control). Existing approaches [3–5,12] to single-stage CC-OPF under DC power flow and Gaussian uncertainties directly formulate the CC-OPF problem in terms of the parameters of the affine feedback, leading to finite-dimensional second-order cone programs. However, the relevance and advantages of non-Gaussian uncertainties for modeling load patterns and renewables have been emphasized in the literature [13–15]. Certain non-Gaussian distributions (such as Beta distributions) allow compact supports and skewed probability density functions, which hence overcome modeling shortcomings of purely Gaussian settings. For example, to model a load

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List of Symbols

N	Number of buses
$\mathcal{N}$	Set of bus indices
N_l	Number of lines
$\mathcal{N}_l$	Set of line indices
u	Controllable active power
d	Uncontrollable active power
p_l	Line power flow
α	AGC coefficients
J	Cost function
$\underline{x}, \overline{x}$	Lower bound, upper bound of x
$\mathbf{1}_n$	n-dimensional column vector of ones
ϕ	Power transfer distribution factor matrix
Ω	Set of outcomes
$\mathbb{P}$	Probability measure
$\mathcal{L}^2(\Omega, \mathbb{R})$	Hilbert space of second-order random variables w.r.t. probability measure $\mathbb{P}$
ξ	Stochastic germ
ψ_ℓ	ℓth basis function
$\boldsymbol{\psi}$	Vectorized basis $\boldsymbol{\psi} = [\psi_1, \dots, \psi_L]^\top$
$\langle \cdot, \cdot \rangle$	Scalar product
$L+1$	PCE dimension
x	Random variable
$\tilde{x} = \mathsf{x}(\tilde{\xi})$	Realization of random variable x
x_ℓ	ℓth vector of PCE coefficients of x
X	Matrix of PCE coefficients of x of degree greater zero
$\mathrm{E}[\mathsf{x}]$	Expected value of x
$\mathrm{Var}[\mathsf{x}]$	Variance of x

via a Gaussian random variable always bears a non-zero probability for the load acting as a producer. Arguably, this probability may be small, but an uncertainty description that rules out this possibility by design is physically consistent and desirable.

We remark that how to address the reformulation of inequality constraints in the problem formulation, i.e. issue (i), is a user-specific choice. As such, this choice resembles a trade-off between computational tractability and modeling accuracy. In contrast, the validity of the power flow equations, i.e. issue (ii) imposes a physical equality constraint that has to be accounted for in the problem formulation. The present paper proposes a general framework for chance-constrained OPF that combines modeling uncertainties in terms of continuous random variables of finite variance, *and* a rigorous mathematical consideration of the power flow equations as equality constraints of the OPF problem. It is shown that a formulation of the CC-OPF problem in terms of random variables naturally leads to engineering-motivated affine policies. Under the mild assumption that uncertainties are modeled as continuous random variables of finite variance with otherwise arbitrary probability distributions, our findings highlight that the optimal affine policies are indeed random-variable minimizers of an underlying CC-OPF problem.

A consequence of the last item is that the proposed general framework to CC-OPF embeds *and* extends current approaches [3–5,12] which consider purely Gaussian settings.

The key step is to formulate the CC-OPF problem rigorously with random variables as decision variables. This unveils the infinite-dimensional nature of CC-OPF. A three-step methodology concisely describes the proposed approach to CC-OPF: formulation, parameterization, optimization. This results in optimal affine policies that satisfy power balance despite uncertainties. The corresponding optimization problem scales well in terms of the number of uncertainties. For common individual chance-constraint reformulations it leads to a second-order cone program. Polynomial chaos expansion (PCE) is employed to represent all occurring random variables by finitely many deterministic coefficients.

While PCE dates back to the late 30s [16], it has been applied to power systems only recently, for example to design a power converter [17], to design observers in the presence of uncertainties [18], and to solve stochastic power flow [19–23]. The applicability of PCE to OPF problems under uncertainty has been demonstrated in [19–22]. The works [21,22] focus on computational details when implementing PCE. In contrast, [19,20] mention that the power flow equations are always satisfied. However, [19,20] do not put PCE approaches to OPF in relation to other existing approaches, and do not show optimality of affine policies. Instead, the present paper takes a different view: starting from existing approaches [3–5,12] we show that PCE is a generalization; the more mathematical nature of PCE is thus related to the engineering practice of affine policies.

The present manuscript focuses on a framework for single-stage OPF problems under uncertainty, highlighting the importance of affine control policies rigorously irrespective of the kind of distribution of the uncertainty. Affine policies have also been applied to multi-stage OPF under uncertainty [8,10,24,25], where their use is motivated based on engineering intuition. For multi-stage OPFproblems the handling of the inequality constraints is similar to single-stage OPF: it comprises analytically reformulated chance constraints [25], convex reformulations [26], and scenario-based approaches [27].

Summing up, the contributions of our work are as follows: We provide a problem formulation of chance-constrained OPF in terms of random variables that is shown to contain existing approaches [3–5,12]. We further give a rigorous proof showing when affine policies are optimal. Additionally, we highlight an important dichotomy: optimal policies of chance-constrained OPF correspond to optimal random variables. Finally, we provide a tractable and scalable reformulation of the random-variable problem in terms of a second-order cone program by leveraging polynomial chaos expansions. The combination of the contributions provide a tractable framework for chance-constrained OPF.

The remainder is organized as follows: Section 2 introduces the CC-OPF problem in terms of random variables, and demonstrates the flexibility of the proposed formulation: existing approaches for Gaussian uncertainties can be obtained as special cases (Section 2.2). The observations at the end of Section 2 lead to a three-step methodology to CC-OPF, presented in Section 4 in greater detail. Section 3 introduces polynomial chaos expansion as a mathematical tool that is required to tackle Section 4. The methodology developed in Section 4 is demonstrated for a tutorial 3-bus example in Section 5.1, and a 300-bus test case in Section 5.2.

2. Preliminaries and problem formulation

Consider a connected N-bus electrical transmission network in steady state that is composed of linear components, for which the DC power flow assumptions are valid (lossless lines, unit voltage magnitude constraints, small angle differences). The N_l lines have indices $\mathcal{N}_l = \{1, \dots, N_l\}$. For simplified presentation each bus $i \in \mathcal{N} = \{1, \dots, N\}$ is assumed to be connected to one generation unit, and one fixed but uncertain power demand/generation. The net active power realization $p \in \mathbb{R}^N$ is $p = u + d$, where $u \in \mathbb{R}^N$ represents adjustable/controllable (generated) power, and $d \in \mathbb{R}^N$ resembles (uncontrollable) power demand in case of $d_i < 0$ for bus $i \in \mathcal{N}$, or (uncontrollable) renewable feed-in in case of $d_j > 0$ for bus $j \in \mathcal{N}$. The goal of (deterministic) OPF is to minimize generation costs $J(u)$ with $J : \mathbb{R}^N \to \mathbb{R}$ such that the power flow equations are satisfied (equality constraints), and generation limits and line flow