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Instrument noise calibration with arbitrary sensor orientations

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ABSTRACT

Instrument noise calibration is indispensable for laboratory and field testing with applications in disciplines such as seismology and structural health monitoring, establishing the basic information for the quality of data. Different methods exist assuming different kinds of information, among which the 'three-channel method' developed by Sleeman and co-workers allows one to calibrate the power spectral density (PSD) of instrument noise without prior information. The method makes use of the sample cross-covariance of three data channels assumed to measure the same input motion. In reality, the input motions of the three channels are never identical due to sensor alignment error and spatial incoherence of the input motion. This leads to bias in the estimated noise PSD, which turns out to also increase with the signal-to-noise ratio. In this paper, the noise calibration problem is investigated analytically to yield explicit formulas that account for the bias due to alignment error and spatial incoherence. Leveraging on fundamental understanding of the bias, a method is proposed which can overcome the bottleneck stemming from alignment error. The proposed method is still based on three collocated sensors but now it makes use of multi-dimensional (biaxial or triaxial) motion data. The latter is the key for the method to be applicable (unbiased) for arbitrary sensor orientations, which significantly enhances the robustness and accuracy of 'huddle test'. Numerical studies with simulated data and a series of specially designed experiments based on servo-accelerometers are presented to verify the analytical findings, to provide a critical appraisal of the proposed method and to demonstrate practical applications.

1. Introduction

Instrument noise calibration aims at determining the in-situ characteristics of the noise of data channels, which is attributed to sensor, data acquisition hardware, etc. Such information is used downstream and is therefore indispensable for many disciplines such as seismology [1–3] and structural health monitoring [4–8], establishing a baseline confidence and precision quantification for measurements. Specification and calibration certificate of instruments can provide nominal information about noise characteristics but they cannot replace in-situ calibration.

By its very nature noise is mixed with the target signal to be measured and neither one is known unless under special circumstances. Noise is commonly modelled by a stationary stochastic process and its strength is characterised by the power spectral density (PSD). An intuitively direct way to extract the noise of an instrument is to eliminate the actual vibration response from the data, which practically requires one to isolate the sensor from fixtures [9,10]. While this may not be always feasible, a common alternative is to perform a 'huddle test' [11–13], where multiple sensors to be calibrated are

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placed together to measure the same input motion. By virtue of redundant information the PSD of instrument noise can be estimated by statistical means. The 'two-channel method' [14] is a technique that uses two collocated and co-aligned data channels. It requires prior information about the transfer functions of the data channels, whose error can smear into the estimated noise characteristics [15,16]. The 'three-channel method' [17] allows one to estimate the PSD of instrument noise without prior information. It has become one of the preferred methods [18] and was applied to calibrate a variety of sensors [19–23].

One pivotal assumption for the two- and three-channel method is that the collocated channels measure the same input motion. For this reason the channels should be oriented along the same direction. In implementation, alignment error is inevitable and it has been found in empirical studies to induce bias in the estimated noise PSD [24–26], which turns out to increase with the signal-to-noise ratio [27,28]. Some attempts [29,30] have been made to mitigate misalignment, although the fundamental issue remains unresolved.

In this work, the three-channel method is first investigated analytically (Section 2) to yield an explicit formula for the statistical bias that allows one to understand its origins. Leveraging on such understanding, a new method based on three collocated sensors but now applicable for arbitrary sensor orientations is developed in Section 3, allowing one to calibrate instruments in a more robust and accurate manner. In addition to alignment error, the effect of spatial incoherence is also investigated in Section 4. A comprehensive study based on synthetic and experimental data is presented in different sections to verify the analytical findings, to provide a critical appraisal of the proposed method and to demonstrate applications. Recommendations for the proposed method are summarised in Section 5.

2. Three-channel method

The three-channel method assumes three collocated and co-aligned channels from three sensors (say Sensors i, j and k) measuring a common 'input motion', which refers to the mechanical motion experienced by the sensor and is the target to be measured. Fig. 1 shows a schematic diagram where the circle represents the sensor and the arrow inside indicates the orientation. Assuming that the input–output relationship of the instrument is linear, the output signal x_i of Sensor i can be modelled in the time domain as the convolution of the input motion z with the impulse response g_i of Sensor i and added with the data channel noise n_i:

$$x_i = g_i * z + n_i \tag{1}$$

where '*' denotes the convolution; the dependence of quantities on time has been omitted for notational simplicity. Similar expressions can be written for other sensors. In the frequency domain, the relationship analogous to (1) is

$$X_i = G_i Z + \varepsilon_i \tag{2}$$

where X_i, G_i, Z and ε_i denote respectively the scaled Fourier transforms (FTs) of x_i, g_i, z and n_i; their dependence on frequency has been omitted for notational simplicity.

Modelling the input motion and instrument noise as stationary stochastic process, the cross PSD between X_i and X_k (say) is equal to $S_{ik} = E(X_i X_k^*)$ where the superscript '*' denotes the complex conjugate and $E(\cdot)$ denotes the expectation. Assuming that the instrument noise between different channels are uncorrelated and that they are also uncorrelated from the input motion, one obtains

$$S_{ii} = G_i S_Z G_i^* + S_{ei} \tag{3}$$

$$S_{ji} = G_j S_Z G_i^* \quad S_{jk} = G_j S_Z G_k^* \quad S_{ik} = G_i S_Z G_k^* \tag{4}$$

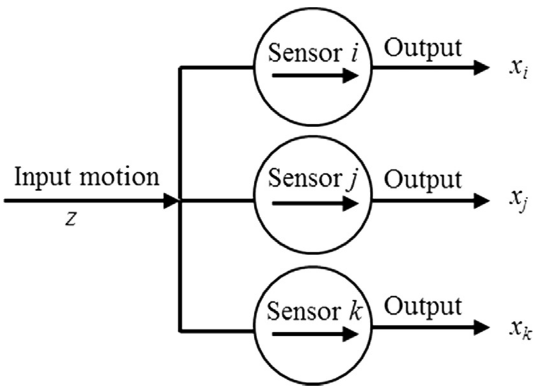

Fig. 1. Schematic diagram of the three-channel method.