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Linear stability of mixed convection in a differentially heated vertical channel filled with high permeable porous-medium

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ABSTRACT

Linear stability analysis is used to analyze the stability of parallel flow induced by an external pressure gradient and buoyancy force in a differentially heated vertical channel filled with a fluid-saturated high permeable porous medium. The non-Darcy model which gives rise to the volume averaged Navier-Stokes (VANS) equation is used except for some comparative study where Darcy model is also used. The investigation is made for a wide range of Prandtl numbers (Pr) that includes mercury, air, water and heavy oils. The spectral method has been adopted to solve the governing equations of the problem. In the entire considered range of Reynolds number (Re) the linear stability results show that the type of instability for mercury is thermal-shear (i.e., the instability is hydrodynamic in origin, resulting from an unstable velocity distribution, and obtains most of its kinetic energy (KE) through shear source), for water as well as heavy oil it is thermal-buoyant (i.e., most of the KE for this instability is obtained through work done by fluctuating buoyant force), whereas for air it is interactive (i.e., KE for this instability is obtained from both shear as well as buoyant sources). When Re is fixed at 1000, the appearance of point of inflection in the basic flow velocity for a fluid with Pr less than 30 acts as a necessary condition for instability for all considered values of Darcy number. Furthermore, the form drag stabilizes the fluid flow with $Pr \geq 7$, however, it acts other way up to a threshold value for fluid flow with $Pr < 1$. Finally, scale analysis reveals that for thermal-shear or interactive instability the minimum critical ΔT for parallel mixed convection flow is less than the same for parallel natural convective flow and it is other way for thermal-buoyant instability. As an example of thermal-buoyant instability, when the channel is filled with a porous medium having permeability $2.5 \times 10^{-6} \mathrm{m}^2$ and half-width 5 cm, the mixed convective flow of water remains stable up to a temperature difference of 13.3°C between the walls, whereas the natural convective flow of water remains stable up to a temperature difference of 8.58°C between the walls.

1. Introduction

It is known that a steady and parallel flow may exist in a vertical channel bounded by impermeable and isothermal parallel planes kept at different temperatures. The flow may be due to the action of buoyancy force which in turn causes natural convection or may be due to the action of both buoyancy force as well as constant pressure gradient along the vertical direction resulting in mixed convection. The former flow is defined by parallel natural convective flow (PNCF) which takes place with zero vertical mass flow rate, whereas the latter one is defined by parallel mixed convective flow (PMCF) which is endowed with nonzero vertical mass flow rate. These types of flow may be configured, with different features, either for a fluid filling the channel or for a fluid-saturated porous slab/channel [1].

A fundamental result of the analysis of natural convection in a vertical porous slab governed by Darcy's law was obtained by Gill [2]. Using linear stability analysis the author has proved that the PNCF in a vertical porous slab is always stable. In spite of this conclusion, further investigations were carried out by other authors. For example, Gill's [2] problem was reinvestigated: using nonlinear analysis by Wolanski [3] and Straughan [4], considering the no-slip condition by Kwok and Chen [5], including the time derivative term in momentum balance equation by Rees [6], studying in a regime of very large DarcyRayleigh numbers by Lewis et al. [7], taking into account the local thermal non-equilibrium between the fluid and solid phases by Rees [8], further through nonlinear stability analysis by Scott and Straughan [9]. These studies are well documented in the papers by Barletta [10,11]. Note that the main motivation for these studies was the suggestion stated by Gill [2] at the end of his paper that possible instability could be obtained by including inertial effects in the local momentum balance equation, i.e.,

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by altering the classical formulation. However, except the work of Kwok and Chen [5] where the basic flow is different from the basic flow of Gill [2], all these investigations finally lead to the basic conclusion reached by Gill [2]. Recently, Barletta, while studying natural convection in a vertical channel [11] and the same as a limiting case of mixed convection [12], has found that Gill's finding does not remain valid even under Darcy model.

Kwok and Chen [5] investigated the effect of no-slip boundary conditions for velocity, implemented by Brinkman's model of momentum balance instead of Darcy's law, and the effect of temperature-dependent viscosity within Darcy's law. In both the cases, they considered a quadratic dependence of the density on temperature. Through linear stability analysis, the authors found that both no-slip conditions and variable viscosity are able to yield instability and, hence, modify the conclusion implied by Gill's proof.

In comparison to the natural convection in a fluid-saturated vertical porous slab, the theoretical investigation of PMCF in a vertical channel filled with porous medium especially when channel's walls are kept at different temperatures is largely overlooked. In the present paper, an attempt is made to understand the stability to small-amplitude perturbations of the PMCF in a differentially heated vertical channel filled with fluid-saturated high permeable porous medium.

Motivation for the present study is based on the following three facts: (i) mixed convection through wall bounded domain filled with porous medium has numerous applications, (ii) stability analysis has not been extended to PMCF in a differentially heated channel, (iii) there are differences between PMCF in a linearly heated channel and differentially heated channel, whose details are narrated below.

Due to the presence of inter-connected voids porous medium has a large surface area to volume fraction and are good candidates for heat transfer enhancement applications. One of the major heat transfer applications is in the electronics industry. Starting with micro-scale (due to the miniaturization of integrated circuits and the assemblage in small volumes) electronics equipment [13] to macro-scale electrical Transformer [14] or giant UPS, a design of good heat transfer equipment has become a challenge to the industry. Use of porous medium such as metal foam has attracted the attention of many researchers due to their desirable flow and thermal characteristics [15]. A metal foam consists of a solid matrix containing a large volume fraction of voids or pores. Open cell metal foams have interconnected voids and are used for heat exchangers, compact electronics cooling, energy absorption, *etc*. For example, in order to exchange the heat of an electronic device from the system to the surrounding, a vertical rectangular duct filled with open cell metal foam can be considered inside the system. The heat generated from the system can be treated as constant heat flux or constant temperature on one of the surfaces of the duct. A steady fluid flow due to an external pressure gradient can be considered through it to exchange the heat from the system to the surrounding. For fast cooling one may enhance the velocity of steady flow, or, increase the gap between the two channel (for a rectangular cylinder) walls. In this situation, steady flow may not remain stable and the exchange of heat from the system to surrounding may be affected due to the mixing of different fluid layers. Therefore, before installing such type of heat exchanger in the system it is essential to understand the fluid flow and heat transfer mechanism through a channel filled with open cell metal foam or high permeable porous medium, especially in the transition state.

Although, several studies in vertical channel/layer have already been made directly or indirectly in this direction but most of these are restricted to laminar fluid flow and heat transfer only (e.g., with uniform heating [16–20], with differentially heating [21–23]), which are well documented in the book by Nield and Bejan [1]. Studies related to transition state are restricted to either linearly heating or imposing constant heat flux condition on the walls. To gain a better perspective of the results to be presented, we summarize their primary conclusions.

A good number of articles [24–30] focus on the linear stability of PMCF due to the linear variation of wall temperature and external pressure gradient to understand the stability of the flow in various aspects. In these studies, the non-Darcy volume-averaged Navier-Stokes (VANS) equation was used and assumed that the solid porous matrix and saturated fluid are in local thermal equilibrium (LTE) state. It has been reported that higher media permeability results in lower stability of the flow, whereas induced form drag stabilizes the flow. Fully developed flow can become unstable under mild heating condition [24]. Furthermore, it has also been pointed out that when buoyancy force acts in the direction of forced flow, three different types of instability, namely shear (or thermal-shear), mixed (or interactive), and buoyant (or thermal buoyant) are possible [25–29]. The type of instability depends on the type of fluid, media permeability, strength of bulk velocity, as well as on induced form drag. In the case when buoyancy force acts in the opposite direction of forced flow (i.e., buoyancy opposed flow) the fully developed flow has two types of instabilities: Rayleigh-Taylor and buoyant (or, thermal buoyant) [27,28]. Note that their study for buoyancy opposed flow was limited to very low permeable porous media. Recently, Bera and Khandelwal [31] have extended the linear stability analysis of above flow with the assumption that the solid porous matrix and saturated fluid are in local thermal non-equilibrium state with buoyancy force in the direction of forced flow. They have found that a higher value of interphase heat transfer coefficient results in more stable flow, i.e., interphase heat transfer coefficient stabilizes the flow. Its stabilizing impact for a fluid with low Prandtl number becomes high when disturbance kinetic energy due to non-isothermal effect is lost to the basic flow. For relatively low permeable medium thermal-buoyant instability is the most dominant instability in the entire range of Prandtl number. Stability of the mixed convection in the same geometry, where buoyant force is induced by symmetric uniform heat flux on the vertical planes, is investigated by Barletta [10]. The details of these studies can be found in Bera and Khandelwal [31].

In comparison to the amount of work done on mixed convection in a linearly heated channel, to the best of our knowledge investigation of the same in a differentially heated channel filled with porous medium is limited to numerical studies by Hadim and Chen [21] as well as Umavathi et al. [23], and experimental study of Pu et al. [22]. Hadim et al. [21] have studied the mixed convective flow in the developing region and have shown that on increasing the Darcy number the distortions in the velocity profile result in an increased velocity near the walls leading to increased heat transfer. Using perturbation method Umavathi et al. [23] have shown that the viscous dissipation enhances the flow reversal in the case of downward flow while it counters the flow in the case of upward flow and the Darcy, as well as inertial drag terms suppress the flow. Pu et al. [22] experimentally found the existence of a secondary convective cell in the mixed-convection regime. Apart from these, Kamath et al. [13] conducted an experimental study of hydraulic performance and heat transfer in flow assisted mixed convection (induced by external pressure gradient and constant heat flux on one vertical side and maintenance of adiabatic condition on the other vertical side) on aluminium metal foams of high porosity. Through the results of the hydraulic experiments, the authors have shown, for the air velocity range used, that the metal foam characteristics deviate from the Darcy flow.

From the above literature review, it is clear that the stability characteristic of PMCF has not been extended to a differentially heated channel. Furthermore, in a linearly heated vertical channel the direction of pressure driven forced flow and the direction of buoyancy force can be either same or opposite, i.e., fully developed parallel flow will be either buoyancy assisted or buoyancy opposed flow. In the case of a differentially heated vertical channel with the pressure driven forced flow from bottom to top, parallel flow can be either buoyancy assisted or assisted near the hot wall but opposed near the cold wall. Therefore, plausible different instability mechanism in a differentially heated channel is expected.

The objective of the present study is two folds. The first is to analyze the stability characteristics of PMCF in a differentially heated channel