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Relay-based hybrid control of minimal-order mechanical systems with applications ☆

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ABSTRACT

This work explores the potential of relay-based control on a one-degree-of-freedom nonlinear mechanical system, in the contexts of both sustaining and damping oscillations. For both cases we state our main results building upon a simple reset formulation (relay feedback) and providing intuitive basic equations from classical mechanics. With a more rigorous description following a hybrid system formalism, we establish then the global asymptotic stability of the corresponding (compact-set) attractors through hybrid Lyapunov tools. The aspects of sustaining and damping oscillation are seen as complementary, because they reduce to a suitable mirroring of the reset surface. Finally, we discuss two applications of our results to the case of a hopping mass and an automotive suspension.

1. Introduction

Linear plants in feedback with nonlinear controllers in the form of relays with hysteresis have been studied since the 1950s (Hamel, 1950; Tsypkin & Herschel, 1958) (and also Tsypkin, 1984) thanks to the favorable features (Atherton, 1996) of the power amplifiers implementing the relays. More generally, at that time it has been recognized with the pioneering work of Clegg (1958) that reset actions in control systems may improve upon the potential of linear designs (this fact was rigorously proven only recently in Beker, Hollot, & Chait, 2001). Follow-up research on reset control comprises the introduction of the concept of First Order Reset Element (FORE) by Horowitz and Rosenbaum (1975), whose use in control systems design in subsequent papers is well summarized in Beker et al. (2001), Beker, Hollot, Chait, and Han (2004), Chait and Hollot (2002) and references therein. In recent years, a revived interest in the field of FORE-based reset control emerged from the availability of new theoretical tools for the analysis of nonlinear hybrid systems (Goebel, Sanfelice, & Teel, 2009, 2012), which motivated more recent works well represented by Baños and Barreiro (2011), Fichera, Prieur, Tarbouriech, and Zaccarian (2016), Satoh (2015) and references therein.

Paralleling the above mentioned research strand, several works such as Åström (1995) (see also Levine, 2010, §18.1.8) have testified the potential of reset control for *sustaining oscillations*, which then extends quite naturally to the problem of generating hybrid limit cycles via hybrid feedback for legged locomotion (Lakatos, Seidel, Friedl, & Albu-Schäffer, 2015; Leach, Gunther, Maheshwari, & Iida, 2014; Reis & Iida, 2014; Yu & Iida, 2014) and for the close setting of juggling systems (Sanfelice, Teel, & Sepulchre, 2007). The potential of applying hybrid dynamical systems techniques in the context of legged locomotion is pointed out in Grizzle, Chevallereau, Sinnet, and Ames (2014), and addressed in technical terms in Teel, Goebel, Morris, Ames, and Grizzle (2013). More

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closely related to this work, the hopping mass is the first milestone in legged locomotion for robots, as witnessed by the impact of the seminal work (Raibert, 1986) (see also the more recent Sayyad, Seth, & Seshu, 2007 and references therein). On a parallel thread, it must be recognized that relay-based feedback has proven to be effective also in *damping oscillations*, which may find applications not only in neuroscience (Montaseri, Javad Yazdanpanah, Pikovsky, & Rosenblum, 2013) but also in the context of semi-active suspensions in the automotive field, as one may appreciate from the survey works in Poussot-Vassal, Spelta, Sename, Savaresi, and Dugard (2012, Section 4) and Savaresi, Poussot-Vassal, Spelta, Sename, and Dugard (2010, Chapter 6).

Motivated by the above observations, in this paper we apply a reset control paradigm to a *nonlinear* mechanical system in order to sustain oscillations and, in a complementary way, to damp them (in the linear case, one of our results relates to Åström, 1995). Furthermore, in the same spirit of Full and Koditschek (1999) and Ghigliazza, Altendorfer, Holmes, and Koditschek (2005) (or Holmes, Full, Koditschek, & Guckenheimer, 2006 for a broader approach encompassing neurobiology and biomechanics), we use minimal order mechanical systems to provide a fundamental explanation to the phenomena of reset-sustained and reset-damped oscillations. Finally, our work is motivated by some newly devised actuators (Leach et al., 2014) for hopping locomotion (Reis & Iida, 2014; Yu & Iida, 2014), whose very fast action resembles the introduction of a "kick" of energy to the mechanical system and can be modeled by a controller reset (an alternative approach close to the nature of this work would be Sanfelice & Teel (2011)). Similar types of actuation are used in Batts, Kim, and Yamane (2016), Lakatos et al. (2015) and in variable impedance actuators (Vanderborght et al., 2013).

In comparison to Baños, Dormido, and Barreiro (2011), Barreiro, Baños, Dormido, and González-Prieto (2014) and Lou, Li, and Sanfelice (2015a, b, 2017), here we consider sustaining and damping oscillations (in a mechanical system) as complementary. The nature of the approach in Lou et al. (2015a, b, 2017) is close to our approach for the case of sustained oscillations, although we do not require any a priori knowledge about the existence of a hybrid periodic solution (as, for instance, in Lou et al., 2015a, Assumption 4.5, item 4)). In Baños et al. (2011) and Barreiro et al. (2014), limit cycles are an undesired dynamics arising when stabilizing through linear resets the origin of a linear system; Barreiro et al. (2014) rely on the matrix exponential for the Poincaré map, and Baños et al. (2011) utilize the approximate method of the describing function (Khalil, 2002, Section 7.2). Our underlying system is instead a *nonlinear* (one-degree-of-freedom) mechanical system; our approach is based on Poincaré map and Lyapunov analysis.

As main contributions, this work focuses on exploring relay-based laws to sustain and damp oscillations for one-degree-of-freedom nonlinear mechanical systems. The analysis is based on hybrid dynamical systems tools (Goebel et al., 2009, 2012), adapted in particular to the study of periodic orbits. The resulting framework is justified and illustrated through two relevant engineering applications. This work extends Bisoffi, Forni, Da Lio, and Zaccarian (2016), whose problem setting originates from Lakatos and Albu-Schäffer (2014), by considering more general nonlinear mechanical systems: the study of sustained oscillations is more detailed and the study of reset-damped oscillations and applications is new. The paper is structured as follows. Sections 2 and 3 present reset-sustained and reset-damped oscillations, respectively. Sections 2.1 and 3.1 provide the main results of the paper and Sections 2.2 and 3.2 discuss technical details and proofs. Our proofs are based on Goebel et al. (2009, 2012) and take inspiration from classical Poincaré analysis (Hirsch & Smale, 1974). Applications are illustrated in Section 4 with a simple model of a hopping robot and with a (semi-active) suspension from the automotive field, respectively.

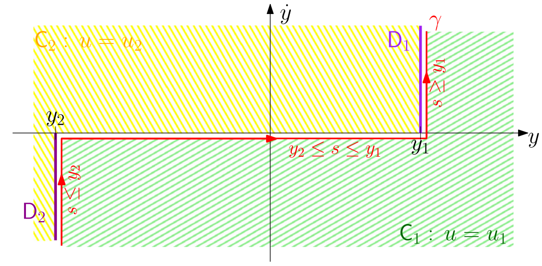

Fig. 1. Curve $s \mapsto \gamma(s)$ in red and related sets C_i and D_i. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Notation. The nonnegative reals and integers are denoted by $\mathbb{R}_{\geq 0}$ and $\mathbb{Z}_{\geq 0}$, respectively. The domain of a function f is denoted by dom f and in the specific case of a solution ϕ to a hybrid dynamical system, dom ϕ denotes a hybrid time domain as in Goebel et al. (2012, Definition 2.3). For a set S, $\overline{S}$ denotes its closure. The closed unit ball is denoted by $\mathbb{B}$.

2. Reset-sustained oscillations

2.1. General theory

Consider the one-degree-of-freedom nonlinear mechanical system

$$m\ddot{y} + c(y, \dot{y})\dot{y} + \frac{\partial U}{\partial y}(y, u) = 0 \tag{1a}$$

where y is the position, $\dot{y}$ is the velocity, $\ddot{y}$ is the acceleration, m is the mass, $c(y, \dot{y})$ is the nonlinear damping coefficient, and $U(y, u)$ is the nonlinear potential whose dependence on the position y is modulated by u, which we use as a control input to the system. We show that system (1a) can be controlled into steady state oscillations by simple, piecewise constant, reset laws.

Based on Fig. 1, consider the curve

$$\gamma(s) := \begin{cases} (y_2, s - y_2) & s \leq y_2 \\ (s, 0) & y_2 \leq s \leq y_1 \\ (y_1, s - y_1) & s \geq y_1 \end{cases} \tag{1b}$$

that divides the plane $(y, \dot{y})$ into the two regions

$$C_1 := \{(y, \dot{y}) : (\dot{y} < 0, y_2 < y \leq y_1) \text{ or } y > y_1\} \tag{1c}$$

$$C_2 := \{(y, \dot{y}) : (\dot{y} > 0, y_2 \leq y < y_1) \text{ or } y < y_2\}, \tag{1d}$$

where $y_1 > y_2$ are two *constant* values. We pursue minimal actuation complexity, so we restrict ourselves to a binary control action u depending on the state $(y, \dot{y})$ as

$$u = \begin{cases} u_1 & \text{if } (y, \dot{y}) \in C_1 \\ u_2 & \text{if } (y, \dot{y}) \in C_2, \end{cases} \tag{1e}$$

where u_1 and u_2 are two *constant* values. At the same time, we also keep a minimal sensing complexity because the resets are triggered when the state $(y, \dot{y})$ is detected to cross the branches

$$D_1 := \{\gamma(s) : s \geq y_1\} \tag{1f}$$

$$D_2 := \{\gamma(s) : s \leq y_2\}. \tag{1g}$$

Models similar to (1) can be found in Lakatos and Albu-Schäffer (2014), Lakatos et al. (2015) and Yu and Iida (2014) wherein the use of such models is extensively motivated in the context of robotic applications. Indeed, we further discuss the practical relevance of this model in Section 4. Solutions to (1) are well defined in view of their hybrid definition that we postpone to Section 2.2. We consider then the following standard concepts.