Automatica 97 (2018) 214–225

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Linear output regulation with dynamic optimization for uncertain linear over-actuated systems[☆]

Matteo Cocetti [c,a,*], Andrea Serrani [b], Luca Zaccarian [c,a]

[a] Department of Industrial Engineering, University of Trento, via Sommarive 9, Trento, IT, 38123, Italy
[b] Department of Electrical and Computer Engineering, 2015 Neil Ave, The Ohio State University, Columbus, OH, 43210, United States
[c] LAAS-CNRS, Université de Toulouse, CNRS, Toulouse, FR, 31077, France

ARTICLE INFO

Article history:
Received 28 June 2017
Received in revised form 12 March 2018
Accepted 20 June 2018

Keywords:
Linear output regulation
Dynamic input allocation
Optimization
Uncertain systems

ABSTRACT

This paper considers the linear output regulation problem for uncertain over-actuated plants. The general form of input redundancy considered in this work implies the existence of multiple control inputs and state trajectories compatible with a prescribed reference for the output. On-line selection, according to certain performance criteria, of the most suitable of these inputs-state trajectories leads to a linear output regulation problem with *dynamic redundancy allocation*. We present a solution that augments the well known *internal model control scheme* with two additional dynamical systems. The first one, named *annihilator*, parametrizes the inputs and the corresponding state trajectories that are invisible from the output. The second one, named *redundancy allocator*, dynamically selects the best solution according to a predefined performance criterion. We derive explicit solutions for the performance criterion equal to relaxed 1, 2, and ∞- norms of the plant input. This set-up is a particular case of the dynamic redundancy allocation problem named *dynamic input allocation*. The proposed solutions can be implemented in an error feedback form and are especially suitable for optimizing sparsity, power and amplitude of the control input. Finally, structural stability, robustness and existence of a unique steady-state are proven.

1. Introduction

Intuitively speaking a system is over-actuated when the number of control inputs is larger than the number of regulated outputs. Over-actuation naturally arises every time there are multiple actuators performing the same action and this is often the case in many engineering applications. The presence of more actuators than strictly necessary could be desirable for many reasons, e.g., safety, fault-tolerant policies, performance or consumption optimization. Popular and well studied examples of over-actuated systems are high performance aircraft (Bodson, 2002) and ships and underwater vehicles (Johansen & Fossen, 2013). However many other application fields are increasing in number, see for instance Zhou, Canova, and Serrani (2016). A large number of actuators introduce a certain *degree of redundancy*, meaning that there exists an entire family of input functions and possibly of state trajectories that are compatible with a prescribed reference for the output (Cristofaro & Galeani, 2014; Galeani, Serrani, Varano, & Zaccarian, 2015; Serrani, 2012; Zaccarian, 2009), or more precisely the system does not have a unique right inverse. This redundancy is exploited to design a *dynamic redundancy allocator* whose primary objective is to dynamically select, among all the feasible inputs, the best one according to a certain cost function.

The setup considered in this work differs from previous studies, notably, Galeani et al. (2015) and Serrani (2012), in several aspects. First, we explicitly take into consideration parametric uncertainties in the plant showing that the problem is structurally well posed. Second, we derive a closed form solution for cost functions involving different norms of the input, solving the so called *dynamic input optimization problem* for three significant cases that involve sparsity, power and amplitude of the plant control signal. The derived strategies only require the tracking error as input. Third, we precisely formulate the dynamic allocation problem within the framework of robust linear output regulation and we show that, in all cases, a dynamic allocator can be designed that provides global exponential stability of the closed-loop system (when the exogenous signals are disconnected), uniform boundedness of all trajectories, and exponential convergence to a unique steady-state by way of a global contractivity property.

☆ This work has been supported in part by NSF under Grant CMMI-1234686 and by CaRiTRo under grant PowerLyap. The material in this paper was partially presented at the 2016 American Control Conference, July 6–8, Boston, MA, USA. This paper was recommended for publication in revised form by Associate Editor Oswaldo Luiz V. Costa under the direction of Editor Richard Middleton.

* Corresponding author at: LAAS-CNRS, Université de Toulouse, CNRS, Toulouse, FR, 31077, France.

E-mail addresses: matteo.cocetti@unitn.it (M. Cocetti), serrani.1@osu.edu (A. Serrani), zaccarian@laas.fr (L. Zaccarian).

https://doi.org/10.1016/j.automatica.2018.08.002
0005-1098/

A preliminary version of this work have been presented in Cocetti, Serrani, and Zaccarian (2016), but here we largely extend the ideas and we present results and proofs in a more general and clear setting. The improvements are essentially threefold. First, we extend the allocation strategy for generic strongly convex objective functions. Second, we provide explicit closed form solutions for three different cost functions of practical interest, i.e., relaxations of the 1, 2 and ∞ norm of the plant input, whereas in Cocetti et al. (2016) only the 2 and ∞ norms were considered. Third, we remove the need for an additional tuning gain required in the solution presented in Cocetti et al. (2016) and we extend the results from semiglobal to global. These latter properties, in turn, have been achieved via the introduction of mild regularity assumptions on the cost function, not required in Cocetti et al. (2016).

The paper is organized as follows: in Section 2 we present the set-up and we formally define the *dynamic allocation problem*. In Section 3 we solve the problem in a fairly general setting. In Section 4 we specialize the findings of Section 3 into the *dynamic input allocation* set up and we provide explicit design for three specific choices of the cost function that are of practical interest, namely relaxed 1, 2, and ∞-norm of the control input. In 5 we present some simulations to show the effectiveness of the proposed techniques. Conclusions are offered in Section 6.

Notation: Let $\mathbb{R}^n$ denote the set of real vectors of dimension n; given a constant $c \in \mathbb{R}$ we write $\mathbb{R}_{\geq c}$ to denote the subset $[c, \infty) \subset \mathbb{R}$. Calligraphic symbols such as $\mathcal{M}$ denote sets, while the formal script font is used to denote real vector spaces locally isomorphic to Euclidean spaces, e.g., $\mathscr{X}$. For a vector $x \in \mathbb{R}^n$, x_i denotes the ith entry, $|x|_1$, $|x|_2$, $|x|_\infty$ are respectively the 1, 2, ∞ norms of x, and $\mathrm{diag}\,(x) \in \mathbb{R}^{n\times n}$ is the diagonal matrix whose ith diagonal element is x_i. Given two vectors $x \in \mathbb{R}^n$, $y \in \mathbb{R}^m$, $\mathrm{col}\,(x, y) := [x^\top, y^\top]^\top \in \mathbb{R}^{n+m}$. For a matrix $M \in \mathbb{R}^{n\times m}$, $M^\top$ denotes its transpose. For square invertible matrices $M \in \mathbb{R}^{n\times n}$, M^{-1} denotes the inverse of M and $M^{-\top}$ its inverse transpose, $M > 0$ $(M \geq 0)$ denotes positive definiteness (semi-definiteness) of M, $\mathrm{spec}(M) = \{\lambda_1, \lambda_2, \ldots, \lambda_n\}$ denotes the spectrum, i.e., the set of the eigenvalues of matrix M, finally $\mathrm{He}\,(M) := (M + M^\top)$ is the Hermitian component of matrix M. If matrix $M \in \mathbb{R}^{n\times n}$ is symmetric the eigenvalues are real and can be always arranged in algebraically non decreasing order as follows $\lambda_1(M) \leq \lambda_2(M) \leq \cdots \leq \lambda_n(M)$. Given $M \in \mathbb{R}^{n\times m}$, M_{ij} denotes the ij component of M with $i = 1, \ldots, n$ and $j = 1, \ldots, m$, while $M^\top_{ij}$ denotes the ij component of $M^\top$ with $i = 1, \ldots, m$ and $j = 1, \ldots, n$. The operator $\mathrm{diag}(M_1, M_2) \in \mathbb{R}^{n\times n}$ denotes the block-wise concatenation of matrices $M_1 \in \mathbb{R}^{n_1\times n_1}$ and $M_2 \in \mathbb{R}^{n_2\times n_2}$ where $n := n_1 + n_2$. Matrix $I_n \in \mathbb{R}^{n\times n}$ denotes the identity matrix of order n but often we will drop the subscript n if the dimension is clear from the context. Given a function $f(x, y)$, $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ we use the following notation: $\nabla_x f(x, y) := \mathrm{col}\left(\frac{\partial f}{\partial x_1}(x, y), \ldots, \frac{\partial f}{\partial x_n}(x, y)\right) \in \mathbb{R}^n$, $\nabla_y f(x, y) := \mathrm{col}\left(\frac{\partial f}{\partial y_1}(x, y), \ldots, \frac{\partial f}{\partial y_m}(x, y)\right) \in \mathbb{R}^m$, $\nabla^2_{yx} f(x, y) := \left(\frac{\partial \nabla_x f}{\partial y_1}(x, y), \ldots, \frac{\partial \nabla_x f}{\partial y_m}(x, y)\right) \in \mathbb{R}^{n\times m}$, and $\nabla^2_x f(x, y) \in \mathbb{R}^{n\times n}$ represents the Hessian matrix with respect x. Finally the symbols $\mathscr{L}_\infty$, $\mathscr{C}^k$, $(k = 0, 1, \ldots)$ denote respectively the set of essentially bounded and k-times differentiable functions.

2. Problem statement

We consider a modified version of the *linear robust output regulation* set up. We start by considering an uncertain plant model of the form,

$$\dot{x}_\mathrm{p} = A_\mathrm{p}(\mu)x_\mathrm{p} + B_\mathrm{p}(\mu)u + P_\mathrm{p}(\mu)w, \tag{1a}$$
$$e = C_\mathrm{p}(\mu)x_\mathrm{p} + Q_\mathrm{p}(\mu)w, \tag{1b}$$

with state $x_\mathrm{p} \in \mathscr{X}_\mathrm{p} \cong \mathbb{R}^{n_\mathrm{p}}$, control input $u \in \mathscr{U} \cong \mathbb{R}^m$ and tracking error $e \in \mathscr{E} \cong \mathbb{R}^p$. The plant matrices $A_\mathrm{p}(\mu) \in \mathbb{R}^{n_\mathrm{p}\times n_\mathrm{p}}$, $B_\mathrm{p}(\mu) \in \mathbb{R}^{n_\mathrm{p}\times m}$, $P_\mathrm{p}(\mu) \in \mathbb{R}^{n_\mathrm{p}\times s}$, $C_\mathrm{p}(\mu) \in \mathbb{R}^{p\times n_\mathrm{p}}$, $Q_\mathrm{p}(\mu) \in \mathbb{R}^{p\times s}$ depend continuously on a vector $\mu \in \mathcal{M}$, whose values are assumed to range over a known compact set $\mathcal{M} \subset \mathbb{R}^{n_\mu}$ containing the origin. Without loss of generality we assume that $\mu = 0$ corresponds to the nominal model. We assume that the plant (1) is driven by an exogenous signal $w \in \mathscr{W} \cong \mathbb{R}^s$ generated by a known *exosystem* of the form,

$$\dot{w} = Sw, \tag{2}$$

where the matrix S is assumed to be semi-simple and such that $\mathrm{spec}(S) \subset \mathbb{C}^0$. This implies that for any initial condition $w(0)$ the arising solution $w(\cdot)$ to (2) is uniformly bounded. Depending on the context, the signal w may represent references and/or disturbances. Stability and regulation for (1) are ensured by a given error feedback controller of the form

$$\dot{x}_\mathrm{c} = A_\mathrm{c}x_\mathrm{c} + B_\mathrm{c}e \tag{3a}$$
$$u_\mathrm{reg} = C_\mathrm{c}x_\mathrm{c}, \tag{3b}$$

with state $x_\mathrm{c} \in \mathscr{X}_\mathrm{c} \cong \mathbb{R}^{n_\mathrm{c}}$ and output $u_\mathrm{reg} \in \mathscr{U}$. The classical formulation of the robust linear output regulation problem is reported in the following Problem 1.

Problem 1 (*Linear Output Regulation Problem*). Given the plant model (1) with exosystem (2) find, if possible, a controller of the form (3) such that:

(1) The closed-loop matrix
$$\begin{bmatrix} A_\mathrm{p}(\mu) & B_\mathrm{p}(\mu)C_\mathrm{c} \\ B_\mathrm{c}C_\mathrm{p}(\mu) & A_\mathrm{c} \end{bmatrix} \tag{4}$$
obtained through the interconnection $u = u_\mathrm{reg}$ is Hurwitz for all $\mu \in \mathcal{M}$.

(2) Solutions of (1)–(3)–(2) originating from any initial condition $(x_\mathrm{p}(0), x_\mathrm{c}(0), w(0)) \in \mathscr{X}_\mathrm{p} \times \mathscr{X}_\mathrm{c} \times \mathscr{W}$ satisfy $\lim_{t\to+\infty} e(t) = 0$, for all $\mu \in \mathcal{M}$.

The solution of Problem 1 relies on the well known "Internal Model Principle", see Francis (1977), and the following assumptions are necessary for its solvability.

Assumption 1 (*Output Regulation Assumptions*).

(1) the pairs $\left[A_\mathrm{p}(\mu), B_\mathrm{p}(\mu)\right]$ and $\left[C_\mathrm{p}(\mu), A_\mathrm{p}(\mu)\right]$ are respectively stabilizable and detectable for all $\mu \in \mathcal{M}$,

(2) the non resonance condition
$$\mathrm{rank}\begin{bmatrix} A_\mathrm{p}(\mu) - \lambda I & B_\mathrm{p}(\mu) \\ C_\mathrm{p}(\mu) & 0 \end{bmatrix} = n_\mathrm{p} + p, \quad \forall\lambda \in \mathrm{spec}(S)$$
holds for all $\mu \in \mathcal{M}$.

Assumption 1 is required for the existence of a stabilizing error-feedback controller of the form (3) that satisfies the internal model property and solves Problem 1. Moreover since we are interested in over-actuated systems, in addition to Assumption 1, following Serrani (2012) and Zaccarian (2009) we make the following characterizing assumptions on the class of systems under investigation:

Assumption 2 (*Over-actuation*). System (1) is over-actuated, that is, $m > p$ and rank $B_\mathrm{p}(\mu) \geq p$, for all $\mu \in \mathcal{M}$. For simplicity, we also assume that rank $C_\mathrm{p}(\mu) = p$, for all $\mu \in \mathcal{M}$.

Assumption 3 (*Nominal Right-invertibility*). The triplet $\left[C_\mathrm{p}(0), A_\mathrm{p}(0), B_\mathrm{p}(0)\right]$ is right-invertible.