Automatica 97 (2018) 64–72

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Distributed learning consensus for heterogenous high-order nonlinear multi-agent systems with output constraints[☆]

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ARTICLE INFO

Article history:
Received 18 April 2017
Received in revised form 25 April 2018
Accepted 30 June 2018

Keywords:
Multi-agent systems
Iterative learning control
Output constraints
Alignment condition
Barrier Lyapunov function

ABSTRACT

This paper considers the learning consensus problem for heterogenous high-order nonlinear multi-agent systems with output constraints. The dynamics consisting of parameterized and lumped uncertainties is different among different agents. To solve the consensus problem under output constraints, two distributed control protocols are designed with the help of a novel barrier Lyapunov function, which drives the control updating and parameters learning. Both convergence analysis and constraint satisfaction are strictly proved by the barrier composite energy function approach. Illustrative simulations are provided to verify the effectiveness of the proposed protocols.

1. Introduction

In the past decades, multi-agent system (MAS) coordination and control problems have attracted much attention from the control community. Much progress has emerged in formation control, synchronization, flocking, swarm tracking, and containment control among others. For these problems, the consensus framework is an effective approach (Cao, Yu, Ren, & Chen, 2013). The setting of a consensus problem involves triple components, namely, agent model, information exchange topology, and distributed consensus algorithm, respectively. For the agent model, the existing results cover single integrator model (Olfati-Saber & Murray, 2004; Ren, Beard, & Atkins, 2007), double integrator model (Hong, Hu, & Gao, 2006; Ren, 2008; Zhang & Tian, 2009), high-order integrator model (Cui & Jia, 2012), linear system (Scardovi & Sepulchre, 2009; Yu & Wang, 2014), and nonlinear system (Chen & Lewis, 2011; Mehrabian & Khorasani, 2016; Mei, Ren, & Ma, 2011). Moreover, the information exchange topology, described by a graph, has been thoroughly developed in the existing literature (Fang & Antsaklis, 2006; Tahbaz-Salehi & Jadbabaie, 2008). Last, the consensus algorithm is important to generate complex group-level behaviors using simple local coordination rules, which are highly related to practical problems (Khoo, Xie, & Man, 2009; Ren & Beard, 2008; Yang, Tan, & Xu, 2013).

Iterative learning control (ILC) is a matured intelligent control technique to achieve high precision tracking performance by the inherent repetition mechanism (Ahn, Chen, & Moore, 2007; Shen & Wang, 2014; Xu, 2011). Therefore, the ILC strategy has been applied for MASs to achieve learning consensus recently. Ahn and Chen (2009) proposed the first result on formation control using the learning strategy. Later, the reports on satellite trajectory-keeping (Ahn, Moore, & Chen, 2010), mobile robots formation (Chen & Jia, 2010), and coordinated train trajectory tracking (Sun, Hou, & Li, 2013) illustrate successful applications of ILC to MASs. For theoretical research, Yang, Xu, Huang, and Tan (2014, 2015) employed the contraction mapping method for convergence analysis of affine nonlinear MASs. The 2D system technique was used to prove the consensus performance in Meng, Jia, and Du (2013, 2015, 2016) and Meng and Moore (2016) for linear systems. The Lyapunov function method was introduced in Li and Li (2013, 2015, 2016) for MASs where agents were of first-order, second-order and high-order models, respectively. Yang and Xu (2016) also provided a composite energy function (CEF) based analysis for networked Lagrangian systems. While various techniques have been developed for the ILC-based MAS consensus, the existing literature mainly focuses on the conventional system setting without any constraint on the system output.

However, when concerning MASs in the real world, it is found that nearly almost all real systems are subject to certain

☆ The paper is supported by National Natural Science Foundation of China (61673045, 61304085), Beijing Natural Science Foundation (4152040) and the China Scholarship Council (201506885019). The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Changyun Wen under the direction of Editor Miroslav Krstic.

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https://doi.org/10.1016/j.automatica.2018.07.030
0005-1098/

constraints. The constraints arise for the output due to various practical limitations and safety considerations. If we ignore such constraints and conduct the conventional control strategy, the system output may be beyond the tolerant range and lead to serious problems. For example, a platoon of auto-vehicles is a typical MAS, in which the vehicles are required to stay in a regulated range and run within the speed limit all the time. Consequently, when updating the control signal, we should always take these constraints into consideration in order to guarantee a safe drive. Otherwise, traffic accidents would arise for the automatic drive if the vehicle is either out of the road range or over the speed limit. Moreover, due to the physical limitation of wireless networks, there usually exists an upper bound of communication bandwidth in MASs; therefore, the output of each agent should fall in a specified range so that the transmitted data would not exceed the maximal bandwidth. In addition, in consideration of implementation cost, simple and cheap measurement devices are widely used in industrial and automation systems, which may only provide a limited measurement range. In such case, the agent output is required not to exceed the range; otherwise, the output is difficult to measure and then the update cannot proceed. From these observations, we note that the output of each agent in a MAS generally has to satisfy certain constraints, which has not been considered in the existing literature. Once the output constraints are required, it is a natural question how to design and analyze the learning update laws for MASs. This problem motivates the research of this paper. In this paper, we try to propose distributed learning protocols to achieve asymptotical consensus along the iteration axis and guarantee the output constraints simultaneously.

To this end, we apply the idea of barrier Lyapunov function (BLF) similar to Jin and Xu (2013) and Xu and Jin (2013) to handle the output constraints problem. Differing from Jin and Xu (2013) and Xu and Jin (2013), we introduce a general type of BLF and apply it to the design of distributed learning protocols for heterogenous high-order nonlinear MASs. In particular, for a MAS where the dynamics of each agent consists of parameterized and lumped uncertainties, we first define a group of auxiliary functions based on the newly introduced BLF and then apply these functions in the design of the protocols. In this paper, two control protocols are designed. The first one introduces sign functions of the involved quantities to regulate control compensation so that the zero-error asymptotical consensus is achieved while satisfying output constraints. However, such protocol may cause chattering problem due to the frequent sign switching. To facilitate practical applications, we further propose the second control protocol, where the sign function is approximated by a hyperbolic tangent function. In such case, we only guarantee the bounded convergence performance; however, we present a precise estimation of the upper bound, which can help to tune the protocol parameters for a specified consensus performance. We note that Li and Li (2013, 2015, 2016) also applied the CEF method for learning consensus problem. Our paper differs from Li and Li (2013, 2015, 2016) in three aspects: (1) we concentrate on the consensus under output constraints and introduce a general BLF; (2) we provide practical alternative of the algorithm implementations; and (3) we employ distinct analysis techniques.

The rest of the paper is arranged as follows. Section 2 proposes the problem formulation and the general barrier Lyapunov function. Section 3 presents two control protocols and the main theorems, whose proofs are put in the Appendix. Section 4 gives illustrative simulations on an engineering system. Section 5 concludes this paper.

Notations: $\mathscr{G} = (\mathscr{V}, \mathscr{E})$ is a weighted graph. $\mathscr{V} = \{v_1, \ldots, v_N\}$ is a nonempty set of nodes/agents, where N is the number of nodes/agents. $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$ is the set of edges/arcs. $(v_i, v_j) \in \mathscr{E}$ indicates that agent j can get information from agent i. $\mathscr{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ denotes the topology of a weighted graph $\mathscr{G}$. a_{ij} is the weighted value, and $a_{ij} = 1$ if $(v_i, v_j) \in \mathscr{E}$, otherwise $a_{ij} = 0$. In addition, $a_{ii} = 0$, $1 \le i \le N$. $d_i = \sum_{j=1}^{N} a_{ij}$ is the in-degree of agent i. $\mathscr{D} = \mathrm{diag}\{d_1, \ldots, d_N\}$ is the in-degree matrix. $\mathscr{L} = \mathscr{D} - \mathscr{A}$ is the Laplacian matrix of a graph $\mathscr{G}$. $\mathscr{N}_i$ denote the set of all neighborhoods of ith agent, where an agent v_j is said to be a neighborhood of agent v_i if v_i can get information from v_j. An agent does not belong to its neighborhood. ε_j denote the access of jth agent to the desired trajectory, that is, $\varepsilon_j = 1$ if agent v_j has direct access to the full information of desired trajectory, otherwise $\varepsilon_j = 0$. $\|\mathbf{x}\|$ denotes the Euclidean norm for a vector $\mathbf{x}$.

2. Problem formulation

Consider a heterogeneous MAS formulated by N ($N > 2$) agents, where the jth agent is modeled by the following high-order nonlinear system

$$\begin{aligned} \dot{x}_{i,j,k} &= x_{i+1,j,k}, \quad i = 1, \ldots, n-1, \\ \dot{x}_{n,j,k} &= \theta_j^T(t)\xi_{j,k}(t) + b_{j,k}(t)u_{j,k} + \eta_{j,k}(t), \\ y_{j,k} &= \{x_{1,j,k}, x_{2,j,k}\}, \end{aligned} \tag{1}$$

where $i = 1, 2, \ldots, n$ denotes the ith dimension of state, $j = 1, 2, \ldots, N$ denotes the agent, and $k = 1, 2, \ldots$ is the iteration number. Denote the state of the jth agent at the kth iteration as $x_{j,k} \triangleq [x_{1,j,k}, \ldots, x_{n,j,k}]^T$. $\theta_j^T(t)\xi_{j,k}(t)$ is the parametric uncertainty, where $\theta_j(t)$ is an unknown parameter vector of the jth agent, which is continuous and bounded on the operation interval $[0, T]$, while $\xi_{j,k}(t) \triangleq \xi_j(x_{j,k}, t)$ is a known time-varying vector-function. $b_{j,k}(t) \triangleq b_j(x_{j,k}, t)$ is the unknown time-varying control gain. $\eta_{j,k}(t) \triangleq \eta_j(x_{j,k}, t)$ is the unknown lumped uncertainty with a known upper bounded function $|\eta_j(x_{j,k}, t)| \le \rho(x_{j,k}, t)$. In the following, denote $\xi_{j,k} \triangleq \xi_j(x_{j,k}, t)$, $b_{j,k} \triangleq b_j(x_{j,k}, t)$, $\eta_{j,k} \triangleq \eta_j(x_{j,k}, t)$, and $\rho_{j,k} \triangleq \rho(x_{j,k}, t)$ where no confusion arises. The system output $y_{j,k} = \{x_{1,j,k}, x_{2,j,k}\}$ can be either $x_{1,j,k}$ or $x_{2,j,k}$ or both, but cannot be varying. For the high-order system, it is required that the outputs should satisfy the given boundedness constraints.

Remark 1. The agent model (1) was also investigated in Li and Li (2016), where the input gain is set to be one and the lumped uncertainty is bounded by a constant. The model (1) for a single system was also considered in Jin and Xu (2013), where the lumped uncertainty is assumed to be variation-norm-bounded. In such case, the tracking reference is assumed to take the same structure of the system model. In this paper, all these requirements are removed. In addition, the model (1) represents a wide range of system uncertainties, as the neural networks and fuzzy approximation-based transformations of general nonlinear systems usually conform to this model.

Let the desired trajectory (virtual leader) be x_r, $x_r \triangleq [x_{1,r}, \ldots, x_{n,r}]^T$ satisfying that $\dot{x}_{i,r} = x_{i+1,r}$, $1 \le i \le n-1$ and $\dot{x}_{n,r} = f(t, x_r)$ with bounded $f(t, x_r)$.

The following assumptions are required for analysis.

- A1 Assume that the input gain $b_{j,k}$ does not change its sign. Meanwhile, it has lower and upper bounds. That is, we assume $0 < b_{\min} \le b_{j,k} \le b_{\max}$, where $b_{\min}$ is known.
- A2 Each agent satisfies the alignment condition, $x_{j,k}(0) = x_{j,k-1}(T)$. In addition, the desired trajectory is spatially closed, that is, $x_r(0) = x_r(T)$.

Remark 2. In the conventional ILC literature, the so-called identical initialization condition (i.i.c.), i.e., $x_{j,k}(0) = x_r(0)$ for all agents and iterations, is the most common assumption for iteration re-initialization. However, this condition is difficult to satisfy for