Automatica 97 (2018) 134–142

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Energy-based feedback control for stochastic port-controlled Hamiltonian systems☆

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ARTICLE INFO

Article history:
Received 12 May 2017
Received in revised form 29 January 2018
Accepted 15 May 2018

Keywords:
Port-controlled Hamiltonian systems
Stochastic dynamical systems
Energy functions
Energy-based control
Stochastic stability

ABSTRACT

In this paper, we develop an energy-based static and dynamic control framework for stochastic port-controlled Hamiltonian systems. In particular, we obtain constructive sufficient conditions for stochastic feedback stabilization that provide a shaped energy function for the closed-loop system while preserving a Hamiltonian structure at the closed-loop level. In the dynamic control case, energy shaping is achieved by combining the physical energy of the plant and the emulated energy of the controller. Several numerical examples are presented that demonstrate the efficacy of the proposed passivity-based stochastic control framework.

1. Introduction

In numerous applications where dynamical system models are used to describe the behavior of natural and engineering systems, stochastic components and random disturbances are typically incorporated into the models. The stochastic aspects of the models are used to quantify system uncertainty and system disturbances as well as the dynamic relationships of sequences of random events between system–environment interactions. In the recent papers by Rajpurohit and Haddad (2016, 2017), the authors extend classical deterministic dissipativity theory (Willems, 1972) to nonlinear stochastic dynamical systems using basic input–output and state properties. Specifically, a stochastic version of dissipativity theory using both an input–output as well as a state dissipation inequality in expectation for controlled Markov diffusion processes is presented.

Dissipativity theory and in particular passivity-based control frameworks for deterministic port-controlled Hamiltonian systems using energy shaping have been developed in the literature. Specifically, Ortega, van der Schaft, and Maschke (1999) and Ortega, van der Schaft, Maschke, and Escobar (2002a, 1999) develop a control design methodology that achieves stabilization via system passivation. In light of the fact that energy notions involving conservation, dissipation, and transport of energy also arise naturally for dissipative diffusion processes, it seems natural that dissipativity theory can play a key role in the control design of stochastic dynamical systems. Specifically, stochastic dissipativity and passivity theory can be used to design feedback controllers that add dissipation and guarantee stability robustness in probability allowing stochastic stabilization to be understood in physical terms.

In this paper, we use the stochastic stability and dissipativity framework developed in Rajpurohit and Haddad (2016, 2017), to extend the deterministic passivity-based control framework for port-controlled Hamiltonian systems of Ortega, van der Schaft and Maschke (1999), Ortega and van der Schaft (2002) and Ortega, van der Schaft, Maschke and Escobar (1999) to nonlinear stochastic port-controlled Hamiltonian systems. Specifically, an energy-based control framework for stochastic port-controlled Hamiltonian systems is developed using a stochastic controller design methodology that achieves stabilization via stochastic system passivation. The interconnection and damping matrix functions of the stochastic port-controlled Hamiltonian system are shaped so that the physical (Hamiltonian) system structure is preserved at the closed-loop level and the closed-loop average energy function is equal to the difference between the average physical energy of the system and the average energy supplied by the controller. Since the Hamiltonian structure is preserved at the closed-loop level, the passivity-based stochastic controller is *robust* with respect to unmodeled passive dynamics. Passivity-based control architectures are extremely appealing since the control action has a clear *physical* energy interpretation, which can considerably simplify controller implementation.

☆ This work was supported in part by the Air Force Office of Scientific Research under Grant FA9550-16-1-0100. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Laura Menini under the direction of Editor Daniel Liberzon.

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https://doi.org/10.1016/j.automatica.2018.07.031
0005-1098/

Finally, we consider energy-based *dynamic* controllers for stochastic port-controlled Hamiltonian systems, wherein energy shaping is achieved by combining the physical energy of the plant and the emulated energy of the feedback controller. For deterministic systems, this approach has been extensively studied by Ortega, Loria, Kelly, and Praly (1995) and Ortega, Loria, Nicklasson, & Sira-Ramirez (1998) to design Euler–Lagrange controllers for potential energy shaping of mechanical systems. The efficacy of the proposed framework is highlighted on several illustrative numerical examples involving an inverted pendulum and a pair of undamped coupled oscillators.

2. Notation, definitions, and mathematical preliminaries

In this section, we establish notation, definitions, and review some basic results on stability of nonlinear stochastic dynamical systems (Arnold, 1974; Khasminskii, 2012; Øksendal, 1995). Specifically, $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}^n$ denotes the set of $n \times 1$ real column vectors, and $\mathbb{R}^{n\times m}$ denotes the set of $n \times m$ real matrices. We write $\mathcal{B}_\varepsilon(x)$ for the *open ball centered* at x with *radius* ε, $\|\cdot\|$ for the Euclidean vector norm or an induced matrix norm (depending on context), $\|\cdot\|_{\mathrm{F}}$ for the Frobenius matrix norm, A^{T} for the transpose of the matrix A, and I_n or I for the $n\times n$ identity matrix.

We define a complete probability space as $(\Omega, \mathcal{F}, \mathbb{P})$, where Ω denotes the sample space, $\mathcal{F}$ denotes a σ-algebra, and $\mathbb{P}$ defines a probability measure on the σ-algebra $\mathcal{F}$; that is, $\mathbb{P}$ is a nonnegative countably additive set function on $\mathcal{F}$ such that $\mathbb{P}(\Omega) = 1$ (Arnold, 1974). Furthermore, we assume that $w(\cdot)$ is a standard d-dimensional Wiener process defined by $(w(\cdot), \Omega, \mathcal{F}, \mathbb{P}^{w_0})$, where $\mathbb{P}^{w_0}$ is the classical Wiener measure (Øksendal, 1995 p. 10), with a continuous-time filtration $\{\mathcal{F}_t\}_{t\geq 0}$ generated by the Wiener process $w(t)$ up to time t. We denote a stochastic dynamical system by $\mathcal{G}$ generating a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ adapted to the stochastic process $x : \overline{\mathbb{R}}_+ \times \Omega \to \mathcal{D}$ on $(\Omega, \mathcal{F}, \mathbb{P}^{x_0})$ satisfying $\mathcal{F}_\tau \subset \mathcal{F}_t$, $0 \leq \tau < t$, such that $\{\omega \in \Omega : x(t,\omega) \in \mathcal{B}\} \in \mathcal{F}_t$, $t \geq 0$, for all Borel sets $\mathcal{B} \subset \mathbb{R}^n$ contained in the Borel σ-algebra $\mathfrak{B}^n$. Here we use the notation $x(t)$ to represent the stochastic process $x(t,\omega)$ omitting its dependence on ω.

Finally, we write $\operatorname{tr}(\cdot)$ for the trace operator, $(\cdot)^{-1}$ for the inverse operator, $V'(x) \triangleq \frac{\partial V(x)}{\partial x}$ for the Fréchet derivative of V at x, $V''(x) \triangleq \frac{\partial^2 V(x)}{\partial x^2}$ for the Hessian of V at x, and $\mathcal{H}_n$ for the Hilbert space of random vectors $x \in \mathbb{R}^n$ with finite average power, that is, $\mathcal{H}_n \triangleq \{x : \Omega \to \mathbb{R}^n : \mathbb{E}[x^{\mathrm{T}}x] < \infty\}$, where $\mathbb{E}$ denotes expectation. For an open set $\mathcal{D} \subseteq \mathbb{R}^n$, $\mathcal{H}_n^{\mathcal{D}} \triangleq \{x \in \mathcal{H}_n : x : \Omega \to \mathcal{D}\}$ denotes the set of all the random vectors in $\mathcal{H}_n$ induced by $\mathcal{D}$. Similarly, for every $x_0 \in \mathbb{R}^n$, $\mathcal{H}_n^{x_0} \triangleq \{x \in \mathcal{H}_n : x \overset{\text{a.s.}}{=} x_0\}$. Furthermore, C^2 denotes the space of real-valued functions $V : \mathcal{D} \to \mathbb{R}$ that are two-times continuously differentiable with respect to $x \in \mathcal{D} \subseteq \mathbb{R}^n$.

Consider the nonlinear stochastic dynamical system $\mathcal{G}$ given by

$$\mathrm{d}x(t) = f(x(t))\mathrm{d}t + D(x(t))\mathrm{d}w(t), \quad x(t_0) \overset{\text{a.s.}}{=} x_0, \quad t \geq t_0, \tag{1}$$

where, for every $t \geq 0$, $x(t) \in \mathcal{H}_n^{\mathcal{D}}$ is a $\mathcal{F}_t$-measurable random state vector, $x(t_0) \in \mathcal{H}_n^{x_0}$, $\mathcal{D} \subseteq \mathbb{R}^n$ is an open set with $0 \in \mathcal{D}$, $w(t)$ is a d-dimensional independent standard Wiener process (i.e., Brownian motion) defined on a complete filtered probability space $(\Omega, \{\mathcal{F}_t\}_{t\geq t_0}, \mathbb{P})$, $x(t_0)$ is independent of $(w(t) - w(t_0))$, $t \geq t_0$, and $f : \mathcal{D} \to \mathbb{R}^n$ and $D : \mathcal{D} \to \mathbb{R}^{n\times d}$ are continuous functions and satisfy $f(x_{\mathrm{e}}) = 0$ and $D(x_{\mathrm{e}}) = 0$ for some $x_{\mathrm{e}} \in \mathcal{D}$. An *equilibrium point* of (1) is a point $x_{\mathrm{e}} \in \mathcal{D}$ such that $f(x_{\mathrm{e}}) = 0$ and $D(x_{\mathrm{e}}) = 0$. It is easy to see that x_{e} is an equilibrium point of (1) if and only if the constant stochastic process $x(\cdot) \overset{\text{a.s.}}{=} x_{\mathrm{e}}$ is a solution of (1).

Here, we assume that $f : \mathcal{D} \to \mathbb{R}^n$ and $D : \mathcal{D} \to \mathbb{R}^{n\times d}$ satisfy the uniform Lipschitz continuity condition

$$\|f(x) - f(y)\| + \|D(x) - D(y)\|_{\mathrm{F}} \leq L\|x - y\|, \quad x, y \in \mathcal{D}, \tag{2}$$

and the growth restriction condition

$$\|f(x)\|^2 + \|D(x)\|_{\mathrm{F}}^2 \leq L^2(1 + \|x\|^2), \quad x \in \mathcal{D}, \tag{3}$$

for some Lipschitz constant $L > 0$, and hence, since $x(t_0) \in \mathcal{H}_n^{\mathcal{D}}$ and $x(t_0)$ is independent of $(w(t) - w(t_0))$, $t \geq t_0$, it follows that there exists a unique solution $x \in \mathcal{L}^2(\Omega, \mathcal{F}, \mathbb{P})$, where $\mathcal{L}^2(\Omega, \mathcal{F}, \mathbb{P})$ denotes the set of equivalence class of measurable and square-integrable $\mathbb{R}^n$ valued random processes on $(\Omega, \mathcal{F}, \mathbb{P})$ over semi-infinite parameter space $[0, \infty)$, to (1) in the following sense. For every $x \in \mathcal{H}_n^{\mathcal{D}} \setminus \{0\}$ there exists $T_x > 0$ such that if $x_1 : [t_0, \tau_1] \times \Omega \to \mathcal{D}$ and $x_2 : [t_0, \tau_2] \times \Omega \to \mathcal{D}$ are two solutions of (1); that is, if $x_1, x_2 \in \mathcal{L}^2(\Omega, \mathcal{F}, \mathbb{P})$ with continuous sample paths almost surely solve (1), then $T_x \leq \min\{\tau_1, \tau_2\}$ and $\mathbb{P}\big(x_1(t) = x_2(t),\ t_0 \leq t \leq T_x\big) = 1$.

The following definitions introducing the notions of Lyapunov and asymptotic stability in probability along with positive invariance of a Borel set with respect to (1) are needed.

Definition 1 (*Khasminskii, 2012*). (*i*) The equilibrium solution $x(t) \overset{\text{a.s.}}{\equiv} x_{\mathrm{e}}$ to (1) is *Lyapunov stable in probability* if, for every $\varepsilon > 0$ and $\rho \in (0, 1)$, there exist $\delta = \delta(\rho, \varepsilon) > 0$ such that, for all $x_0 \in \mathcal{B}_\delta(x_{\mathrm{e}})$,

$$\mathbb{P}^{x_0}\left(\sup_{t\geq t_0} \|x(t) - x_{\mathrm{e}}\| > \varepsilon\right) \leq \rho. \tag{4}$$

(*ii*) The equilibrium solution $x(t) \overset{\text{a.s.}}{\equiv} x_{\mathrm{e}}$ to (1) is *asymptotically stable in probability* if it is Lyapunov stable in probability and there exists $\delta > 0$ such that if $x_0 \in \mathcal{B}_\delta(x_{\mathrm{e}})$, then

$$\lim_{x_0\to x_{\mathrm{e}}} \mathbb{P}^{x_0}\left(\lim_{t\to\infty} \|x(t) - x_{\mathrm{e}}\| = 0\right) = 1. \tag{5}$$

(*iii*) The equilibrium solution $x(t) \overset{\text{a.s.}}{\equiv} x_{\mathrm{e}}$ to (1) is *globally asymptotically stable in probability* if it is Lyapunov stable in probability and, for all $x_0 \in \mathbb{R}^n$,

$$\mathbb{P}^{x_0}\left(\lim_{t\to\infty} \|x(t) - x_{\mathrm{e}}\| = 0\right) = 1. \tag{6}$$

Definition 2 (*Mao, 1999*). An open set $\mathcal{D} \subset \mathbb{R}^n$ is said to be *positively invariant with respect to* (1) if it is Borel and, for all $x_0 \in \mathcal{D}$, $\mathbb{P}^{x_0}\left(x(t) \in \mathcal{D}\right) = 1,\ t \geq t_0$.

Finally, we provide sufficient conditions for local and global asymptotic stability in probability for the nonlinear stochastic dynamical system (1). First, however, recall that the *infinitesimal generator* $\mathcal{L}$ of $x(t)$, $t \geq 0$, with $x(0) \overset{\text{a.s.}}{=} x_0$, is defined by

$$\mathcal{L}V(x_0) \triangleq \lim_{t\to 0^+} \frac{\mathbb{E}^{x_0}[V(x(t))] - V(x_0)}{t}, \quad x_0 \in \mathcal{D}, \tag{7}$$

where $\mathbb{E}^{x_0}$ denotes the expectation with respect to the transition probability measure $\mathbb{P}^{x_0}(x(t) \in \mathcal{D}) \triangleq \mathbb{P}(t_0, x_0, t, \mathcal{D})$ (Øksendal, 1995 Def. 7.7). If $V \in \mathrm{C}^2$ and has a compact support, and $x(t)$, $t \geq 0$, satisfies (1), then the limit in (7) exists for all $x \in \mathcal{D}$ and the infinitesimal generator $\mathcal{L}$ of $x(t)$, $t \geq 0$, can be characterized by the system *drift* and *diffusion* functions $f(x)$ and $D(x)$ defining the stochastic dynamical system (1) and is given by (Øksendal, 1995 Thm. 7.9)

$$\mathcal{L}V(x) \triangleq \frac{\partial V(x)}{\partial x} f(x) + \frac{1}{2}\operatorname{tr} D^{\mathrm{T}}(x)\frac{\partial^2 V(x)}{\partial x^2} D(x), \quad x \in \mathcal{D}. \tag{8}$$

Theorem 3 (*Khasminskii, 2012, Thm. 5.3, Cor. 5.1, Thm. 5.11*). *Consider the nonlinear stochastic dynamical system* (1) *and assume that there exists a two-times continuously differentiable function* $V : \mathcal{D} \to \mathbb{R}$ *such that*

$$V(x_{\mathrm{e}}) = 0, \tag{9}$$