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Brief paper

Monte Carlo localisation of a mobile robot using a Doppler–Azimuth radar[☆]

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ABSTRACT

This paper investigates the moving robot localisation problem using a Doppler–Azimuth radar array. The solution is formulated in the framework of nonlinear/non-Gaussian estimation using a particle filter and a random finite set (RFS) model of measurements. The proposed approach assumes the availability of a feature-based map, radar measurements and robot odometry data. The associations between the measurements and the features of the map (landmarks) are unknown. The RFS model is adopted to deal with false and missed detections and uses Murty's algorithm to reduce computation when solving the association problem. The proposed particle filter incorporates the Kullback–Leibler Distance (KLD)-Sampling to reduce computational time. Monte-Carlo simulation results demonstrate the efficacy of the proposed algorithm.

1. Introduction

Successful deployment of a mobile robot requires the ability to localise and orient itself as it moves through an environment. In this paper we consider robotic applications where satellite navigation is not available, such as indoor, underground, undersea or extraterrestrial environments. Furthermore, the sensor available for robot navigation is a Doppler radar. Since frequency measurements can be easily obtained at low cost and with high accuracy, Doppler radars have the advantage over traditional robot navigation sensors: they are cheaper and lighter than LIDAR sensors, and potentially have a greater range than both LIDAR and ultrasound sensors. However, using Doppler shifts for navigation is difficult, because of the poor information content of such measurements.

In the past, Doppler-shift frequencies have been used for the purpose of target localisation and tracking (Battistelli, Chisci, Fantacci, Farina, & Graziano, 2015; Ristic & Farina, 2013; Shames, Bishop, Smith, & Anderson, 2013). The problem of robot navigation using Doppler-shift frequencies is novel and very different from target tracking. The robot, equipped with a Doppler radar, has to estimate its own position and heading (also known as the robot pose) using the measured Doppler-shifts. The Doppler-shift on the transmitted radar wave occurs due to the movement of a robot relative to the static features (landmarks) at known locations. The paper builds on our previous work (Guan, Ristic, Wang, Moran, & Evans, 2016). In particular, it relaxes the following assumptions made in Guan et al. (2016): knowledge of the robot's initial pose distribution, perfect Doppler radar detection and known landmark to measurement associations.

The main feature of the proposed method is that it is based on the particle filter and models the measurements at each scan as random finite sets (Mahler, 2007). A collective term for particle-filter based robot navigation algorithms is Monte Carlo localisation (MCL) (Kootstra & De Boer, 2009; Liu, Shi, Zhao, & Xu, 2008; Thrun, Fox, Burgard, & Dellaert, 2001). The novelty of our MCL algorithm is that it is using realistic Doppler radar measurements, characterised by false and missed detections as well as poor accuracy of azimuth measurements. The RFS modelling allows us to formulate robot localisation as a particle filter which can elegantly deal with false and missed radar detections. RFS models have previously been used for feature-based simultaneous localisation and mapping (SLAM) using range-azimuth measurements (Adams, Vo, Mahler, & Mullane, 2014). While in this paper we focus on robot localisation only (i.e. the feature-based map is assumed known), the solution is proposed for less informative measurements of Doppler-shifts and azimuth. The theoretical formulations of a general RFS particle filter can be found in Ristic (2013) and Vo, Singh, and Doucet (2005).

The RFS particle filter that we propose incorporates Murty's algorithm (Murty, 1968) along with the Kullback–Leibler Distance

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(KLD)-Sampling algorithm (Fox, 2003) to reduce computational time. The assumptions are that a feature-based map is known and that the robot odometry data are available. In summary, this paper includes the following contributions: (a) application of RFS particle filters to mobile robot localisation on a feature-based map with a Doppler radar; (b) realistic Doppler radar measurements modelled with false and missed-detections and unknown measurement-to-landmark association; (c) incorporation of KLD-Sampling with RFS particle filters.

The remainder of this paper is organised as follows. Section 2 introduces the robot motion model and the sensor measurement model. Section 3 proposes a likelihood function based on RFS modelling, followed by the RFS particle filter with KLD sampling. Numerical simulation studies are presented in Section 4 and conclusions are drawn in Section 5.

2. Mathematical models

2.1. Robot motion model

The robot's pose at discrete-time k is a vector $\mathbf{x}_k = [x_k, y_k, \theta_k]^T$, where (x_k, y_k) are Cartesian coordinates of robot location and θ_k is its heading. Let the control input applied from $k-1$ to k be comprised of a translational and rotational velocity and denoted $\mathbf{u}_k = [v_k, w_k]^T$. Robot motion is described by equation (Thrun, Burgard, & Fox, 2006, Ch.7): $\mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_k) + \mathbf{e}_k$, where $\mathbf{e}_k \sim \mathcal{N}(\mathbf{0}_{(3\times1)}, \mathbf{Q}_k)$ is the Gaussian process noise with zero mean and the covariance matrix $\mathbf{Q}_k$, and $\mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_k)$ is a three-dimensional vector with the following components:

$$f_1(\mathbf{x}_{k-1}, \mathbf{u}_k) = x_{k-1} - \frac{v_k}{w_k}[\sin\theta_{k-1} - \sin(\theta_{k-1} + w_k\Delta t)]$$
$$f_2(\mathbf{x}_{k-1}, \mathbf{u}_k) = y_{k-1} + \frac{v_k}{w_k}[\cos\theta_{k-1} - \cos(\theta_{k-1} + w_k\Delta t)]$$
$$f_3(\mathbf{x}_{k-1}, \mathbf{u}_k) = \theta_{k-1} + w_k\Delta t$$

$\Delta t = t_k - t_{k-1}$ is the sampling interval. The covariance matrix $\mathbf{Q}_k$ is approximated as (Thrun et al., 2006, Ch.7):

$$\mathbf{Q}_k \approx \mathbf{B}_k\mathbf{D}_k\mathbf{B}_k^T \tag{1}$$

where diagonal matrix $\mathbf{D}_k = \mathrm{diag}(\gamma_1 v_k^2 + \gamma_2 w_k^2, \gamma_3 v_k^2 + \gamma_4 w_k^2)$ is introduced to model noisy perturbations to the commanded velocities. The parameters $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ are selected by the user to reflect the model inaccuracies in how the control vector affects robot's motion. Matrix $\mathbf{B}_k$ maps the noise from the control space to the state space, which is defined as

$$\mathbf{B}_k = \left.\frac{\partial\mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_k)}{\partial\mathbf{u}_k}\right|_{\mathbf{x}_{k-1},\mathbf{u}_k} = \left.\begin{bmatrix} \frac{\partial f_1}{\partial v_k} & \frac{\partial f_1}{\partial w_k} \\ \frac{\partial f_2}{\partial v_k} & \frac{\partial f_2}{\partial w_k} \\ \frac{\partial f_3}{\partial v_k} & \frac{\partial f_3}{\partial w_k} \end{bmatrix}\right|_{\mathbf{x}_{k-1},\mathbf{u}_k} \tag{2}$$

where:

$$\frac{\partial f_1}{\partial v_k} = \frac{1}{w_k}\{\sin(\theta_{k-1} + w_k\Delta t) - \sin\theta_{k-1}\}$$
$$\frac{\partial f_1}{\partial w_k} = \frac{v_k}{w_k^2}(\sin\theta_{k-1} - \sin(\theta_{k-1} + w_k\Delta t) + w_k\Delta t\cos(\theta_{k-1} + w_k\Delta t))$$
$$\frac{\partial f_2}{\partial v_k} = \frac{1}{w_k}\{\cos\theta_{k-1} - \cos(\theta_{k-1} + w_k\Delta t)\}$$
$$\frac{\partial f_2}{\partial w_k} = \frac{v_k}{w_k^2}(-\cos\theta_{k-1} + \cos(\theta_{k-1} + w_k\Delta t) + w_k\Delta t\sin(\theta_{k-1} + w_k\Delta t))$$
$$\frac{\partial f_3}{\partial v_k} = 0; \quad \frac{\partial f_3}{\partial w_k} = \Delta t$$

The transition density can then be expressed as $\pi(\mathbf{x}_k|\mathbf{x}_{k-1}, \mathbf{u}_k) = \mathcal{N}(\mathbf{x}_k; \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_k), \mathbf{Q}_k)$.

2.2. Doppler radar measurement model

The (known) feature-based two-dimensional map $\mathbf{m}$ consists of the Cartesian coordinates of M landmarks $\mathbf{m}^{(i)} = \left[x^{(i)}, y^{(i)}\right]^T$, for $i = 1, 2, \ldots, M$. The cardinality of a measurement set at time k is random, because the probability of landmark detection is less than one, and spurious (false) detections (clutter) are possible. Let the measurement set at time k be denoted $\mathbf{Z}_k = \left[\mathbf{z}_{k,1}, \ \mathbf{z}_{k,2}, \ \ldots, \mathbf{z}_{k,L_k}\right]^T$. Thus each measurement vector $\mathbf{z} \in \mathbf{Z}_k$ consists of the Doppler-shift frequency and the azimuth angle, either as a return from a landmark or as a false detection. Let us assume that clutter is a Poisson point process whose statistics are constant both in space and time (for simplicity): its average rate is λ and its spatial distribution is $c(\mathbf{z})$. Furthermore, let the probability of detection of a landmark $\mathbf{m}^{(j)}$ be $P_d(\mathbf{m}^{(i)}) \leq 1$, for $i = 1, \ldots, M$. Specifically, the probability of detection is modelled as

$$P_d(\mathbf{m}^{(i)}) = e^{-\beta\times M_d{}^2} \tag{3}$$

where M_d is the distance between the robot and the landmark and β is a design parameter (a constant). As the distance M_d increases, the probability of the detection $P_d(\mathbf{m}^{(i)})$ decreases.

The measurement equation for a true radar return $\mathbf{z}_{k,j} \in \mathbf{Z}_k$ from a landmark $\mathbf{m}^{(i)}$ is given by

$$\mathbf{z}_{k,j} = h(\mathbf{x}_k, \mathbf{m}^{(i)}) + \mathbf{n}_k \tag{4}$$

where $h(\mathbf{x}_k, \mathbf{m}^{(i)}) = \left[d_k^{(i)}, \ \phi_k^{(i)}\right]^T$, and

$$d_k^{(i)} = -\frac{2f_c}{c}\frac{v_k\left\{(x_k - x^{(i)})\cos\theta_k + (y_k - y^{(i)})\sin\theta_k\right\}}{\sqrt{(x_k - x^{(i)})^2 + (y_k - y^{(i)})^2}} \tag{5}$$

$$\phi_k^{(i)} = \arctan\frac{y^{(i)} - y_k}{x^{(i)} - x_k} - \theta_k \tag{6}$$

with f_c being the radar carrier frequency and c the speed of light. Noise $\mathbf{n}_k$ in (4) is assumed to be white zero-mean Gaussian, uncorrelated to $\mathbf{e}_k$. Its covariance matrix is $\mathbf{R}$. The likelihood function of a true radar detection $\mathbf{z}_{k,j} \in \mathbf{Z}_k$ of a landmark $\mathbf{m}^{(i)}$, based on (4), is $g(\mathbf{z}_{k,j}|\mathbf{x}_k, \mathbf{m}^{(i)}) = \mathcal{N}(\mathbf{z}_k; h(\mathbf{x}_k, \mathbf{m}^{(i)}), \mathbf{R})$.

3. The proposed solution

3.1. Theoretical framework

Recursive pose estimation can be carried out in the Bayesian framework through the prediction and update steps. Suppose the posterior probability density function (PDF) of pose at time $k-1$ is $p(\mathbf{x}_{k-1}|\mathbf{Z}_{1:k-1}, \mathbf{u}_{1:k-1})$, where $\mathbf{x}_{1:k-1} \equiv \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{k-1}$ and so forth. The prediction equation, using $\mathbf{u}_k$ and the transition density, is given by

$$p(\mathbf{x}_k|\mathbf{Z}_{1:k-1}, \mathbf{u}_{1:k}) = \int \pi(\mathbf{x}_k|\mathbf{x}_{k-1}, \mathbf{u}_k)p(\mathbf{x}_{k-1}|\mathbf{Z}_{1:k-1}, \mathbf{u}_{1:k-1})d\mathbf{x}_{k-1} \tag{7}$$

while the update equation, using the measurement set $\mathbf{Z}_k$, applies the Bayes rule:

$$p(\mathbf{x}_k|\mathbf{Z}_{1:k}, \mathbf{u}_{1:k}) = \frac{\varphi(\mathbf{Z}_k|\mathbf{x}_k, \mathbf{m})\cdot p(\mathbf{x}_k|\mathbf{Z}_{1:k-1}, \mathbf{u}_{1:k})}{\int\varphi(\mathbf{Z}_k|\mathbf{x}_k, \mathbf{m})\cdot p(\mathbf{x}_k|\mathbf{Z}_{1:k-1}, \mathbf{u}_{1:k})d\mathbf{x}_k} \tag{8}$$