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Brief paper

Robust fault tolerant explicit model predictive control ☆

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ABSTRACT

In this study, a new algorithm for explicit model predictive control of linear discrete-time systems subject to linear constraints, disturbances, uncertainties, and actuator faults is developed. The algorithm is based on dynamic programming, constraint rearrangement, multi-parametric programming, and a solution combination procedure. First of all, the dynamic programming is used to recast the problem as a multi-stage optimization problem. Afterwards, the constraints are rearranged in an innovative manner to take into account the worst admissible situation of unknown bounded disturbances, uncertainties, and actuator faults. Then, the explicit solution of the reformulated optimization problem for each stage is obtained using the multi-parametric programming approaches. Finally, a recursive procedure for combination and substitution of the solutions is presented to extract the desired explicit control law.

1. Introduction

Nowadays, considering the fact that the modern industries are getting more complicated, the use of automated control systems is necessary to optimize their performance. In these systems, an actuator/sensor fault can bring about irrecoverable damages. Moreover, the occurrence of disturbances and uncertainties is inevitable. Therefore, a control system should be designed with effective robustness against the disturbances and uncertainties as well as sufficient tolerance against the faults. A control system maintaining the overall stability along with an acceptable performance level in faulty conditions is called a fault tolerant control (FTC) system.

FTC methods are classified as active (AFTC) or passive (PFTC). AFTC refers to controllers equipped with fault detection and isolation (FDI) module in order to contract the fault effect intelligently (Blanke, Kinnaert, Lunze, Staroswiecki, & Schröder, 2006). Whereas, PFTC immunizes the system against presumed faults, and hence, it is fundamentally based on the robust control theory (for example, see Benosman & Lum, 2010). Upon occurrence of a fault in AFTC, the FDI module detects it, the controller reconfigures correspondingly, and then the control system switches to the new configuration (for example, see Fawzi, Tabuada, & Diggavi, 2014; Teixeira, Shames, Sandberg, & Johansson, 2015). This procedure, however, results in three challenges: (a) the required time postpones the execution of an appropriate action, (b) its performance highly depends on the FDI accuracy, and (c) switching to a new configuration can cause unusual transients shocking the system (Jiang & Yu, 2012). In contrast, PFTC has a fixed simple configuration with no need to FDI. Consequently, it responds as fast as possible with no switching transients (Jiang & Yu, 2012). However, since it makes the system robust against the worst possible condition, PFTC is more conservative compared to AFTC.

Different methods have been proposed for FTC, but to the best of our knowledge, no one has addressed the passive FTC using the model predictive control (MPC). The MPC is one of the most useful model based control strategies which can deal with multi-variable cases with input/state constraints (Camacho & Alba, 2013). This method employs a model to predict the future process behavior and calculates an optimal control input sequence at each time step through the optimization of an objective function. Only the first input is applied to the process, and the procedure is repeated at the next sampling instant with the newly updated states. The major problem of the MPC is heavy online computational requirements which limits its widespread application. To resolve this issue, the explicit MPC (eMPC) has been proposed (see Bemporad, Morari, Dua, & Pistikopoulos, 2002; Borrelli, Bemporad, & Morari, 2017; TøNdel, Johansen, & Bemporad, 2003), which computes the explicit state feedback solution of a finite horizon linear quadratic optimal control problem with input/state constraints. The eMPC utilizes the multi-parametric programming to calculate the control law as an explicit piecewise affine function of state and reference vector (Pistikopoulos, Georgiadis, & Dua, 2007a). In this method, the closed form of the optimal solution is obtained offline. Hence,

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the online computational effort is considerably reduced and limited to finding the region of the current states. This advantage, along with the traditional MPC advantages made eMPC controllers applicable to fast processes and agile systems to which traditional MPC controllers were not useful at all. Faísca, Kouramas, Saraiva, Rustem, and Pistikopoulos (2008) and Kouramas, Faísca, Panos, and Pistikopoulos (2011) have employed the dynamic programming along with the multi-parametric programming to recast and solve the original MPC problem as a multi-stage optimization problem. Solving the reformulated problem instead of the original one considerably decreases the computational costs, especially for large scale problems with larger prediction horizons.

Generally, the nominal MPC controllers cannot guarantee the feasibility, i.e. the constraint satisfaction, when the disturbances and model uncertainties affect the system (Pistikopoulos, 2009). Therefore, the robust MPC methods have been developed (e.g. see Kothare, Balakrishnan, & Morari, 1996; Mayne, Seron, & Raković, 2005). In contrast to the robust MPC, the current literature on the robust eMPC is not rich at all (Pistikopoulos, 2009). Grancharova and Johansen (2003) derived the robust explicit control law for a linear quadratic MPC problem with unknown bounded additive disturbances as well as linear inequality constraints by the rearrangement of the constraints for the worst admissible situation. Sakizlis, Kakalis, Dua, Perkins, and Pistikopoulos (2004) solved the same problem with a closed loop formulation to decrease the conservativeness. The sub-optimal robust explicit control law calculation for a constrained linear system with uncertain model matrices and a quadratic objective function were also investigated in some other studies (Kouramas, Panos, Faísca, & Pistikopoulos, 2013; Pistikopoulos, Faísca, Kouramas, & Panos, 2009). In these papers, the same approach as proposed in Faísca et al. (2008) based on dynamic programming was implemented. To the best of our knowledge, no paper is available in the eMPC literature on design of a robust controller against the disturbances and uncertainties together. Furthermore, the eMPC has not been utilized in the FTC literature to date.

In this work, the disturbances, model uncertainties, and actuator faults are jointly incorporated in an MPC regulation problem on a linear system thereby a robust fault tolerant explicit control law is derived under state/input constraints. The algorithm is proposed in a novel framework, innovatively employing the presented ideas in the works of Grancharova and Johansen (2003); Kouramas et al. (2011, 2013). In this approach, the dynamic programming along with the multi-parametric programming is utilized based on work of Kouramas et al. (2011); whereas constraint rearrangement is inspired by Grancharova and Johansen (2003); Kouramas et al. (2013). The remainder of this paper is organized as follows. The MPC problem is introduced in Section 2. The next section presents how to obtain the robust fault tolerant explicit control law followed by an illustrative example. A brief discussion on the concluding remarks is presented in the last section.

2. Problem formulation

Consider the linear discrete-time system

$$x_{k+1} = Ax_k + Bu_k + Td_k \tag{1}$$

where $x_k \in \mathbb{R}^n$, $u_k \in \mathbb{R}^m$, and $d_k \in \mathbb{R}^s$ are the state, control input and disturbance vectors, respectively, subject to constraints $x_k \in \mathcal{X}$, $u_k \in \mathcal{U}$, and $d_k \in \mathcal{D}$. The sets $\mathcal{X} = \{x \in \mathbb{R}^n | Mx \leq \mu\}$ and $\mathcal{U} = \{u \in \mathbb{R}^m | Nu \leq \gamma\}$ are convex polyhedral with $M \in \mathbb{R}^{n_M \times n}$, $\mu \in \mathbb{R}^{n_M}$, $N \in \mathbb{R}^{m_N \times m}$, and $\gamma \in \mathbb{R}^{m_N}$. Furthermore, the unknown disturbance d_k is limited to $d_i^L \leq d_{k,i} \leq d_i^U$ $(i = 1, \ldots, s)$. The system matrices A, B are uncertain with unknown bounded matrices of ΔA, ΔB defined as

$$A = A_0 + \Delta A\,,\ \ \Delta A \in \mathbb{R}^{n\times n}\,,\ \ -\epsilon_A|A_0| \leq \Delta A \leq \epsilon_A|A_0| \tag{2}$$

$$B = B_0 + \Delta B\,,\ \ \Delta B \in \mathbb{R}^{n\times m}\,,\ \ -\epsilon_B|B_0| \leq \Delta B \leq \epsilon_B|B_0| \tag{3}$$

in which, A_0, B_0 are the nominal matrices and $\epsilon_A, \epsilon_B \in [0, 1)$. The element-wise absolute value of the nominal matrices denoted by $|X|$ is also defined as $\{|x_{ij}|\}$ where $X = \{x_{ij}\}$.

Let $u_{k,i}$ stand for the ith actuator at time instant k and $u_{k,i}^F$ denote its faulty form modeled as

$$u_{k,i}^F = (1 - \alpha_{k,i})u_{k,i}\,,\ \ 0 \leq \alpha_{k,i} \leq \alpha_i^U \leq 1 \tag{4}$$

where $\alpha_{k,i}$ and α_i^U indicate the corresponding failure percentage at time instant k and its upper bound, respectively. Therefore, $\alpha_{k,i} = 0$ represents the healthy condition, while $\alpha_{k,i} = 1$ corresponds to its complete loss. Now, we define $\Gamma_k \triangleq I_m - \alpha_k$ where $\alpha_k \triangleq diag\{\alpha_{k,1}, \ldots, \alpha_{k,m}\}$ and I_m is the identity matrix of size m. Hence, $u_k^F = \Gamma_k u_k$ and the system, under the effect of actuators failure, is described by

$$x_{k+1} = Ax_k + B\Gamma_k u_k + Td_k \tag{5}$$

The constrained MPC problem is defined as

$$\min_{\mathbf{U_t}} \quad \sum_{k=0}^{N-1}[\bar{x}_{k|t}^T Q\bar{x}_{k|t} + u_{k|t}^T Ru_{k|t}] + \bar{x}_{N|t}^T P\bar{x}_{N|t} \tag{6}$$

$$\begin{aligned}
\text{subj. to:}\quad & \bar{x}_{k+1|t} = A_0\bar{x}_{k|t} + B_0u_{k|t}\,,\ k = 0, \ldots, N-1 && (7)\\
& x_{k+1|t} = Ax_{k|t} + B\Gamma_k u_{k|t} + Td_{k|t}\,,\ k = 0, \ldots, N-1 && (8)\\
& x_{k|t} \in \mathcal{X}\,,\ k = 0, \ldots, N,\\
&\qquad \forall\, d_{0|t}, \ldots, d_{k-1|t} \in \mathcal{D},\\
&\qquad \forall\, \Delta A_{0|t}, \ldots, \Delta A_{k-1|t}\ \textit{which satisfy}\ (2),\\
&\qquad \forall\, \Delta B_{0|t}, \ldots, \Delta B_{k-1|t}\ \textit{which satisfy}\ (3),\\
&\qquad \forall\, \alpha_{0|t}, \ldots, \alpha_{k-1|t}\ \textit{which satisfy}\ (4) && (9)\\
& u_{k|t} \in \mathcal{U}\,,\ k = 0, \ldots, N-1 && (10)\\
& x_{N|t} \in \mathcal{X}_f = \{x \in \mathbb{R}^n | M_f x \leq \mu_f\},\\
&\qquad \forall\, d_{0|t}, \ldots, d_{N-1|t} \in \mathcal{D},\\
&\qquad \forall\, \Delta A_{0|t}, \ldots, \Delta A_{N-1|t}\ \textit{which satisfy}\ (2),\\
&\qquad \forall\, \Delta B_{0|t}, \ldots, \Delta B_{N-1|t}\ \textit{which satisfy}\ (3),\\
&\qquad \forall\, \alpha_{0|t}, \ldots, \alpha_{N-1|t}\ \textit{which satisfy}\ (4). && (11)
\end{aligned}$$

where the objective function (6) involves predicted states of the nominal system, $\bar{x}_{k|t}$, whose evolution is given in (7). The dynamics of the system, which captures the effect of all uncertain parameters, is given in (8), and in (9)–(11) the constraints for all realization of the uncertainties are imposed. Furthermore, $\mathbf{U_t} \triangleq \{u_{0|t}, \ldots, u_{N-1|t}\}$ is the control input sequence, $x_{0|t} = x_t$ is the optimization parameter, Q and P are symmetric positive semi-definite, R is symmetric positive definite, $\mathcal{X}_f$ is the terminal set, and N is the prediction horizon. At any time instant, the newly updated optimization parameter, i.e. x_t, is used to extract new control input sequence $\mathbf{U_t}$. For simplicity, in the remainder of the paper, subscript index k is used instead of subscript index $k|t$.

At each sampling time, it is desired to obtain the current optimal control signal u_0^* as an explicit function of x_0, which satisfies the constraints (9)–(11) for all admissible values of disturbances, uncertainties, and actuators failure. To apply multi-parametric programming, the objective function is assumed to penalize the behavior of the nominal system (Kouramas et al., 2013). We call the closed-form solution of this problem as the robust fault tolerant explicit control law.