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Brief paper

Constrained evolutionary games by using a mixture of imitation dynamics ☆

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ABSTRACT

Game dynamics have been widely used as learning and computational tool to find evolutionarily stable strategies. Nevertheless, most of the existing evolutionary game dynamics, i.e., the replicator, Smith, projection, Brown–Von Neumann–Nash, Logit and best response dynamics have been analyzed only in the unconstrained case. In this work, we introduce novel evolutionary game dynamics inspired from a combination of imitation dynamics. The proposed approach is able to satisfy both upper- and lower-bound constraints. Moreover, dynamics have asymptotic convergence guarantees to a generalized-evolutionarily stable strategy. We show important features of the proposed game dynamics such as the positive correlation and invariance of the feasible region. Several illustrative examples handling population state constraints are provided.

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1. Introduction

The evolutionary game dynamics have become a powerful tool in the modeling of strategic interactions (Hofbauer & Sigmund, 1988, 2013; Sandholm, 2010). Recently, this approach has been implemented in many engineering applications (Quijano et al., 2017). For instance, this game theoretical approach has been used in drinking water networks (Barreiro-Gomez, Quijano, & Ocampo-Martinez, 2016), wireless networks (Lasaulce & Tembine, 2011; Tembine, Altman, El-Azouzi, & Hayel, 2010), multiple access control (Zhu, Tembine, & Başar, 2013), congestion games (Poveda, Brown, Marden, & Teel, 2017; Sandholm, 2010), temperature control (Obando, Pantoja, & Quijano, 2014), among others. From the perspective of evolutionary games, constraints and convergence to a generalized-Nash equilibrium have not been studied so far. Nevertheless, most of the emerging engineering applications involving resource allocation problems require the consideration of multiple physical and/or operational constraints. Therefore, there is still need to study the population-games approach under interaction or migration restrictions, but also with constraints over the variables, which is the main issue discussed in this paper.

The main contribution of this paper consists on novel constrained evolutionary game dynamics able to reach a generalized-Nash equilibrium. To this end, we present the design of both centralized and distributed evolutionary game dynamics under individual constraints. The deduction of the aforementioned dynamics is obtained from the mean dynamics (Barreiro-Gomez, Obando, & Quijano, 2017; Sandholm, 2010; Tembine, Altman, ElAzouzi, & Sandholm, 2008) and by designing appropriately the switching rates, i.e., by introducing some new modified revision protocols. Hence, we show that the novel resulting evolutionary population dynamics are a mixture of imitative classical dynamics. Furthermore, the features of the constrained evolutionary game dynamics are formally presented, e.g., the positive correlation satisfaction, the invariance of the simplex set, and the asymptotic stability of the generalized-Nash equilibrium (generalized evolutionary stable strategy) under both full-potential games and non-full-potential games with monotone decreasing fitness functions. In addition, a way to determine the uniqueness of the equilibrium point under the proposed dynamics is discussed by means of the Kellogg's fixed point theorem, different from the monotonicity condition of the fitness functions or by using contractive mapping properties. Finally, we present some illustrative examples, including an optimization-based engineering application, allowing to evidence the suitable performance of the proposed dynamics for some imposed constraints.

This paper is organized as follows. Section 2 presents the preliminary concepts of evolutionary games. Section 3 introduces the proposed novel evolutionary-game dynamics and the formal analysis corresponding to their relevant features. Finally,

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Sections 4 and 5 present some illustrative examples and conclusions, respectively.

Notation. Let $\mathbb{R}$, $\mathbb{R}_{>0}$, $\mathbb{R}_{\geq 0}$ denote the set of real, positive real, and non-negative real numbers, respectively. Moreover, $\mathbb{1}_n$ ($\mathbf{0}_n$) denotes the vector with n unitary (null) entries, and $\mathbb{I}_n$ denotes the $n \times n$ identity matrix. Boundary of the set Δ is denoted by $\partial\Delta$. Finally, consider the operator $[\cdot]_+ = \max(0, \cdot)$, and the spectrum of a matrix $M \in \mathbb{R}^{n\times n}$ denoted by $\lambda(M)$.

2. Preliminaries

Consider a large population of agents selecting an available strategy from the set $\mathcal{S} = \{1, \ldots, n\}$, being $n \in \mathbb{Z}_{>0}$. The scalar $x_i \in \mathbb{R}_{\geq 0}$ corresponds to the proportion of agents selecting the strategy $i \in \mathcal{S}$. Hence, let $x \in \mathbb{R}^n_{\geq 0}$ be a population state or strategic distribution throughout the strategies. The set representing the possible set of population states such that the population mass remains constant is given by the simplex set

$$\Delta = \left\{ x \in \mathbb{R}^n_{\geq 0} : \sum_{i\in\mathcal{S}} x_i = 1 \right\}.$$

Agents make decisions in order to maximize their utilities, which are determined by a fitness function $f_i : \Delta \to \mathbb{R}$, i.e., it takes a population state and return a reward for the proportion of agents x_i selecting the strategy $i \in \mathcal{S}$. Similarly, the function $f : \Delta \to \mathbb{R}^n$ corresponds to the population fitness function.

Definition 1. A population game is $f : \Delta \to \mathbb{R}^n$. ■

In a population game denoted by f, given that $x \in \Delta$, we equivalently interpret $x(t)$ as a *mixed strategy* used by the players at time t, i.e., players select the action $i \in \mathcal{S}$ with probability $x_i(t)$ (Lasaulce & Tembine, 2011). The objective in the population is to reach a Nash equilibrium.

Definition 2 (*Nash equilibria*)**.** The set of Nash equilibria of the population game f is given by

$$\mathrm{NE}(f) = \left\{ x \in \Delta : x \in \arg\max_{y\in\Delta} y^\top f(x) \right\}. \quad ■$$

If the population game $f : \Delta \to \mathbb{R}^n$ is continuous, then there is at least a Nash equilibrium.

Definition 3 (*Games with Monotone Fitness*)**.** The population game $f : \Delta \to \mathbb{R}^n$ is monotone decreasing if

$$(x - y)^\top (f(x) - f(y)) \leq 0, \ \forall\, x, y \in \Delta.$$

Alternatively, if f is continuously differentiable and monotone decreasing, then a sufficient condition is $Df(x) \preceq 0$. ■

Definition 4 (*Full-Potential Game*)**.** f is a full-potential game if there exists a continuously differentiable potential function $g : \mathbb{R}^n \to \mathbb{R}$, such that $f(x) = \nabla g(x)$, $\forall x \in \mathbb{R}^n$. ■

Different from the classic population games (Sandholm, 2010), now we consider that each strategy has an associated carrying capacity, i.e., the constant parameter $\bar{x}_i$ denotes the carrying capacity associated to the strategy $i \in \mathcal{S}$. Hence, let $\underline{x}_i$ denote the minimum limit for the proportion of agents selecting $i \in \mathcal{S}$. Consequently, each proportion of agents is constrained to an interval $x_i \in \mathcal{X}_i = [\underline{x}_i, \bar{x}_i]$, where $\underline{x}_i, \bar{x}_i \in \mathbb{R}_{\geq 0}$, for all $i \in \mathcal{S}$. It follows that $\mathcal{X} = \prod_{i\in\mathcal{S}} \mathcal{X}_i$, and the feasible set of population states is given by $\Delta \cap \mathcal{X}$. In order to simplify the notation, let us consider the vectors $\hat{x} = \bar{x} - x$ and $\check{x} = x - \underline{x}$ representing a tolerance from the actual population state x and the borders $\bar{x}$ and $\underline{x}$. Under this novel perspective, the proportion of agents x_i makes decisions in order to maximize their benefits determined by a fitness function $f_i : \Delta \cap \mathcal{X} \to \mathbb{R}$, being $f : \Delta \cap \mathcal{X} \to \mathbb{R}^n$ the population fitness function. Therefore, the population objective becomes to reach a generalized-Nash equilibrium as defined next.

Definition 5 (*Generalized-Nash equilibria*)**.** Consider each proportion of agents constrained in $x_i \in \mathcal{X}_i = [\underline{x}_i, \bar{x}_i]$, i.e., $x \in \mathcal{X} = \prod_{i=1}^n \mathcal{X}_i$. The set of generalized-Nash equilibria of the population game f is given by

$$\mathrm{GNE}(f, \mathcal{X}) = \left\{ x \in \Delta \cap \mathcal{X} : x \in \arg\max_{y\in\Delta\cap\mathcal{X}} y^\top f(x) \right\}. \quad ■$$

Notice that, if $\underline{x}_i < \bar{x}_i \leq 1$, and $\underline{x}^\top \mathbb{1}_n < 1 < \bar{x}^\top \mathbb{1}_n$, then the set $\Delta \cap \mathcal{X}$ is non-empty, convex and compact. If in addition $f : \Delta \to \mathbb{R}^n$ is continuous, then there exists at least one Generalized-Nash equilibrium. Together with the individual constraints given by the sets $\mathcal{X}_i$, for all $i \in \mathcal{S}$, there are further constrained associated to possible interaction within the population. Let us suppose that the interactions are represented by an undirected and connected graph $\mathcal{G} = (\mathcal{S}, \mathcal{E}, A)$, where $\mathcal{E} \subseteq \{(i,j) : i, j \in \mathcal{S}\}$ is the set of links representing the possible interaction among the proportion of agents, i.e., if $(i,j) \in \mathcal{E}$, then the proportion of agents x_i and x_j can interact to each other. In other words, $(i,j) \in \mathcal{E}$ means that agents selecting the strategy $i \in \mathcal{S}$ could migrate to strategy $j \in \mathcal{S}$ and vice versa. Moreover, $A \in \{0, 1\}^{n\times n}$ is the adjacency matrix of the graph $\mathcal{G}$, and whose entries are $a_{ij} = 1$, if $(i,j) \in \mathcal{E}$; and $a_{ij} = 0$, otherwise. The function $\varrho : \Delta \times \mathbb{R}^n \to \mathbb{R}^{n\times n}_{\geq 0}$ is the revision protocol, which describes how agents are making decisions. The revision protocol takes a population state and the corresponding fitness, and returns a non-negative matrix. Therefore, let $\varrho_{ij}(x, f)$ be the switching rate from the ith to jth strategy. Then, the agents selecting the strategy $i \in \mathcal{S}$ have incentives to migrate to the strategy $j \in \mathcal{S}$ only if $\varrho_{ij}(x, f) > 0$, and it is also possible to design switch rates depending on the topology describing the migration constraints, i.e., $\varrho_{ij}(x, f, A)$, where A is the adjacency matrix of $\mathcal{G}$. The evolutionary game dynamics emerge from the combination of the switching rates imposed by revision protocols and the mean dynamics (Barreiro-Gomez et al., 2017; Sandholm, 2010; Tembine et al., 2008) i.e.,

$$\dot{x}_i = \sum_{j\in\mathcal{S}} x_j \varrho_{ji}(x, f) - x_i \sum_{j\in\mathcal{S}} \varrho_{ij}(x, f), \ \forall i \in \mathcal{S}. \tag{1}$$

There is a class of dynamics, known as imitation dynamics, which has been recently studied, e.g., (Govaert, Ramazi, & Cao, 2017; Zino, Como, & Fagnani, 2017) . Within this class, we present four different *Pairwise Proportional Imitation* revision protocols that allow to consider individual constraints in the evolutionary game dynamics, i.e,

• *Smith-replicator-based Pairwise Proportional Imitation:* Agents imitate the opponent only if its payoff is greater than his own, and if there are options to imitate according to the individual carrying capacity corresponding to the opponent's strategy, i.e.,

$$\varrho^{\mathrm{sr}}_{ij}(x, f) = \frac{1}{x_i} [\hat{x}_j]_+ \cdot [\check{x}_i]_+ \cdot [f_j - f_i]_+.$$

Recall that $\hat{x} = \bar{x} - x$ and $\check{x} = x - \underline{x}$, and notice that, when $\underline{x}_i = 0$, then the revision protocol becomes $\varrho^{\mathrm{sr}}_{ij}(x, f) = [\hat{x}_j]_+ \cdot [f_j - f_i]_+$.

• *Projection-mixture-based Pairwise Proportional Imitation:* Agents imitate the opponent if it is obtaining a higher payoff. Also, this protocol verifies the individual carrying capacity corresponding to the opponent's strategy, i.e.,

$$\varrho^{\mathrm{pm}}_{ij}(x, f) = \frac{1}{nx_i} [\hat{x}_j]_+ \cdot [f_j - f_i]_+.$$

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