Automatica 97 (2018) 278–285

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Stability of continuous-time positive switched linear systems: A weak common copositive Lyapunov functions approach[☆]

Yong Xu[a], Jiu-Gang Dong[b], Renquan Lu[a,*], Lihua Xie[c]

[a] *The School of Automation, Guangdong University of Technology, and Guangdong Key Laboratory of IoT Information Technology, Guangzhou 510006, China*
[b] *Department of Mathematics, Harbin Institute of Technology, Harbin 150001, China*
[c] *School of Electrical and Electronic Engineering, Nanyang Technological University, 639798, Singapore*

ARTICLE INFO

Article history:
Received 3 September 2017
Received in revised form 6 January 2018
Accepted 27 March 2018

Keywords:
Switched systems
Positive systems
Weak Lyapunov functions
Stability

ABSTRACT

In this paper, we study the problem of asymptotic stability of continuous-time positive switched linear systems under both arbitrary and restricted switchings. It is well-known that asymptotic stability under arbitrary switching can be implied by several classes of strong common copositive Lyapunov functions (CLFs), i.e., functions whose derivative along the nontrivial system trajectories is negative. However, asymptotically stable positive switched systems may not admit strong common CLFs. The main contribution of this paper is to study the stability problem by requiring only weak common CLFs. Firstly, necessary and sufficient conditions are established for asymptotic stability under arbitrary switching. Among them, an easily verifiable graphical stability criterion, based on the connectivity of the digraphs associated with the subsystem matrices, is proposed. Secondly, we further relax the obtained graphical condition to derive a relaxed weak excitation condition for asymptotic stability under dwell-time switching. Finally, two examples are provided to illustrate the effectiveness of our theoretical results.

1. Introduction

A switched system consists of two parts—a finite family of systems and a switching signal that selects an active subsystem from the family at every instant of time. Due to their broad applications in engineering, switched systems have attracted a growing interest over the past few decades (Lin & Antsaklis, 2009; Shorten, Wirth, Mason, Wulff, & King, 2007; Sun & Ge, 2011).

Positive switched linear systems (PSLSs), as a special type of switched systems, have great significance from both practical and theoretical points of view. Indeed, positive systems arise in the modeling of practical problems from diverse areas, e.g., biology, economics, communication and engineering control (Hernandez-Vargas, Colaneri, & Middleton, 2013; Shorten, Wirth, & Leith, 2006; Zappavigna, Charalambous, & Knorn, 2012). Meanwhile, the positivity constraint, on the one hand, requires resorting to less settled approaches based on cones and polytopes, and on the other hand, brings about very fruitful but elegant theory (Blanchini, Colaneri, & Valcher, 2015; Li, Lam, Wang, & Date, 2011; Shen & Lam, 2016). Stability property of PSLSs has been a main issue attracting considerable attention of researchers (Fainshil, Margaliot, & Chigansky, 2009; Gurvits, Shorten, & Mason, 2007; Liu & Dang, 2011; Zhao, Zhang, Shi, & Liu, 2012). So far, two stability issues are addressed in the literature, i.e., stability under arbitrary switching and stability under restricted switching. For the stability analysis problem, the first question is whether PSLSs are asymptotically stable when there is no restriction on the switching signals. This problem is usually called stability analysis under arbitrary switching. On the other hand, we know that stability under arbitrary switching is a very strong property. PSLSs may fail to preserve stability under arbitrary switching, but may be asymptotically stable under some restricted switching signals. In this case, most attention has been drawn to identifying classes of stabilizing switching signals, which is often called stability analysis under restricted switching.

Lyapunov functions are central tools in the study of the stability analysis problem. Generally, Lyapunov functions can be classified into strong and weak types, depending on whether their existence is enough to ensure asymptotic stability (strong) or just stability (weak). In particular, when dealing with positive systems, we only

☆ This work was supported in part by the National Natural Science Foundation of China under Grants (61425009, U1611262, 61320106010), the Fundamental Research Funds for the Central Universities (2017FZA5010), the Science and Technology Planning Project of Guangdong Province (2017B010116006), and Guangdong Province Higher Vocational Colleges & Schools Pearl River Young Scholar approved in 2017. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor James Lam under the direction of Editor Ian R. Petersen.

* Corresponding author.
E-mail addresses: yxu@gdut.edu.cn (Y. Xu), jgdong@hit.edu.cn (J.-G. Dong), rqlu@gdut.edu.cn (R. Lu), elhxie@ntu.edu.sg (L. Xie).

https://doi.org/10.1016/j.automatica.2018.04.037
0005-1098/© 2018 Published by Elsevier Ltd.

need Lyapunov functions to be copositive, i.e., taking positive values only in the positive orthant. As a consequence, strong common copositive Lyapunov functions (CLFs) are widely used to deal with the asymptotic stability of PSLSs under arbitrary switching. Three classes of strong common CLFs are reported in the literature. To be specific, necessary and sufficient conditions for the existence of common linear CLFs have been obtained, see, e.g., Ding, Shu, and Liu (2011), Fornasini and Valcher (2010, 2012), Knorn, Mason, and Shorten (2009), Mason and Shorten (2007) and Wu and Sun (2013). It is shown that the existence of a common linear CLF implies that of a common quadratic CLF (Blanchini et al., 2015). Pastravanu and Matcovschi (2014) further explored the existence of common max-type CLFs, which is proved to be independent of the existence of common linear CLFs. However, as we all know, the existence of such strong common CLFs is only a sufficient condition for the asymptotic stability of PSLSs under arbitrary switching. Therefore, studying the stability via such strong common CLFs may sometimes lead to conservatism. In addition, in certain case, the constituent systems of a PSLS cannot share a strong common CLF but a weak one, such as linear compartmental switched systems (Valcher & Zorzan, 2016) and formation control systems (Rantzer, 2012). It is clear that the existence of weak common CLFs is not sufficient for the asymptotic stability. A key question arising here is that, under the existence of weak common CLFs, what additional conditions are needed to ensure the asymptotic stability of PSLSs under arbitrary switching. This issue is our primary goal.

Motivated by the discussions above, in this paper we investigate the stability problem of continuous-time PSLSs in the framework of weak common linear (resp. max-type) CLFs. More precisely, first of all, attention is focused on the asymptotic stability of the considered systems under arbitrary switching, and necessary and sufficient stability criteria are derived. Among them, a novel graphical condition is given in terms of the connectivity of the digraphs associated with the subsystem matrices, which is very easy to check. Then, we loosen the obtained graphical stability condition to ensure the asymptotic stability of the considered systems under dwell-time switching signals. Such stabilizing condition is referred to as the relaxed weak excitation due to the fact that it relaxes the weak excitation introduced in Meng, Xia, Johansson, and Hirche (2017). Finally, two examples are provided to illustrate our results established in this paper.

The rest of this paper is organized as follows. The problem under consideration is formulated in Section 2. The stability analysis under arbitrary switching is provided in Section 3. Based on the graphical stability criterion obtained in Section 3, the relaxed weak excitation is derived in Section 4. Two examples are provided in Section 5 to illustrate the effectiveness of the obtained results. Finally, we conclude this paper in Section 6 and the Appendix contains a technical lemma needed in Section 4.

We close this section with some notation used in this paper. Denote by $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ the n-dimensional real space and the space of $n \times n$ real matrices, respectively. Let $\mathbb{N}$ be the set of natural numbers including zero. The ith component of a vector $\boldsymbol{v}$ is denoted by v_i. For two vectors $\boldsymbol{v}, \boldsymbol{w} \in \mathbb{R}^n$, we write $\boldsymbol{v} \succeq \boldsymbol{w}$ if $v_i \geq w_i$ for all $i \leq n$ and $\boldsymbol{v} \succ \boldsymbol{w}$ if $v_i > w_i$ for all $i \leq n$. The notation $A^\top$ ($\boldsymbol{v}^\top$) stands for the transpose of a matrix A (a vector $\boldsymbol{v}$). A *Metzler matrix* is a real square matrix, whose off-diagonal entries are all nonnegative. We say that a matrix $A \in \mathbb{R}^{n\times n}$ is nonnegative if all of its entries are nonnegative. Denote by $\mathbf{e}_i$ the ith canonical basis vector in $\mathbb{R}^n$, i.e., the vector with all entries equal to zero but the ith one equal to 1.

For a matrix $A = [a_{ij}]_{n\times n}$, its induced digraph is defined by $\mathcal{G}(A) = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, \ldots, n\}$ is the set of vertices and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is the set of arcs so that $(j, i) \in \mathcal{E}$ if and only if $a_{ij} \neq 0$. A *path* in $\mathcal{G}(A)$ from i_0 to i_k is a sequence of distinct vertices $i_0, i_1 \ldots i_k$ such that each successive pair of vertices is an arc of the digraph. The integer k (i.e., the number of its arcs) is the length of the path. If there is a path from i to j, we say that i can reach j, denoted by $i \rightsquigarrow j$. We say that $\mathcal{G}(A)$ is *strongly connected* if each vertex is reachable from the other for any two distinct vertices. In this case, the matrix A is said to be *irreducible*.

2. Problem formation

Consider the continuous-time positive switched linear system:

$$\dot{\boldsymbol{x}}(t) = A_{\sigma(t)}\boldsymbol{x}(t), \quad t \geq 0 \tag{1}$$

where $\boldsymbol{x}(t) \in \mathbb{R}^n_+$ is the system state and $\sigma(t) : [0, \infty) \to \mathcal{P} = \{1, \ldots, m\}$ is a piecewise constant switching signal, continuous from the right, specifying which subsystem is activated at each time t. The switching instants form a strictly increasing sequence $\{t_k\}_{k=0}^{\infty}$ with $\lim_{t\to\infty} t_k = \infty$. The matrices $A_1, \ldots, A_m$ are assumed to be Metzler, which amounts to saying that system (1) is a positive switched linear system, meaning that its solution $\boldsymbol{x}(t) \succeq \mathbf{0}$ for all $t \in [0, \infty)$ if the initial condition $\boldsymbol{x}(0) \succeq \mathbf{0}$.

Our standing assumption is the following.

Assumption 1. There exists a vector $\boldsymbol{v} \succ 0$ such that $\boldsymbol{v}^\top A_p \preceq \mathbf{0}^\top$ for all $p \in \mathcal{P}$.

A particularly important class of systems, which satisfies Assumption 1, is called compartmental systems in the literature, see. e.g., Valcher and Zorzan (2016).

Letting $V(\boldsymbol{x}) = \boldsymbol{v}^\top\boldsymbol{x}$, we see that for any solution $\boldsymbol{x}(t) \succeq \mathbf{0}$ of system (1), $\dot{V}(\boldsymbol{x}(t)) = \boldsymbol{v}^\top A_{\sigma(t)}\boldsymbol{x}(t) \leq 0$ for all $t \geq 0$, namely $V(\boldsymbol{x})$ is a weak common linear copositive Lyapunov function (WCLCLF) for system (1) if and only if Assumption 1 holds. Similarly, if $A_p^\top$ satisfies Assumption 1 for all $p \in \mathcal{P}$, define $V(\boldsymbol{x}) = \max_{i\leq n}\frac{x_i}{v_i}$. By letting $\mathcal{M}(t) = \{i : x_i(t)/v_i = V(\boldsymbol{x}(t))\}$, we have

$$\begin{aligned}
D^+V(\boldsymbol{x}(t)) &= \max_{i\in\mathcal{M}(t)} \frac{\dot{x}_i(t)}{v_i} \\
&= \max_{i\in\mathcal{M}(t)} \frac{\sum_{j=1}^n a^{ij}_{\sigma(t)}x_j(t)}{v_i} \\
&\leq \max_{i\in\mathcal{M}(t)} \frac{\sum_{j=1}^n a^{ij}_{\sigma(t)}v_j(t)V(\boldsymbol{x}(t))}{v_i} \\
&= \max_{i\in\mathcal{M}(t)} \frac{(A_{\sigma(t)}\boldsymbol{v})_iV(\boldsymbol{x}(t))}{v_i} \\
&\leq 0,
\end{aligned}$$

where the first inequality is due to the fact that $a^{ij}_{\sigma(t)}x_j(t) \leq a^{ij}_{\sigma(t)}v_jV(\boldsymbol{x}(t))$ for all $j \neq i \in \mathcal{M}(t)$ and $a^{ii}_{\sigma(t)}x_i(t) = a^{ii}_{\sigma(t)}v_iV(\boldsymbol{x}(t))$ for $i \in \mathcal{M}(t)$. That is, $V(\boldsymbol{x})$ is a weak common max-type copositive Lyapunov function (WCMCLF) for system (1) if and only if Assumption 1 is satisfied with $A_p^\top$ for all $p \in \mathcal{P}$.

We introduce the notion of asymptotic stability for system (1) under arbitrary switching. It is well-known that, for switched linear systems, asymptotic stability is equivalent to exponential stability (Sun & Ge, 2011, Proposition 2.13).

Definition 1. We say that system (1) is asymptotically stable under arbitrary switching if, for every switching signal $\sigma(\cdot)$ and every nonnegative initial condition $\boldsymbol{x}(0)$, the trajectory $\boldsymbol{x}(t)$ converges to zero as $t \to \infty$.

Likewise, the asymptotic stability under restricted switching can be defined by restricting the switching signals to some admissible sets.

We see that the previous two classes of weak Lyapunov functions only guarantee that system (1) is stable, but not asymptotically stable. This paper is devoted to the problem of finding