Automatica 97 (2018) 306–315

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Global stabilization of discrete-time multiple integrators with bounded and delayed feedback[☆]

Bin Zhou *, Xuefei Yang

Center for Control Theory and Guidance Technology, Harbin Institute of Technology, Harbin, 150001, China

ARTICLE INFO

Article history:
Received 12 December 2017
Received in revised form 11 April 2018
Accepted 6 July 2018

Keywords:
Global stabilization
Multiple integrators
Discrete-time systems
Bounded controls
Delayed controls

ABSTRACT

This paper investigates the global stabilization problem of discrete-time multiple integrators with bounded and delayed feedback. In the absence of input delay, two classes of nonlinear feedback laws are proposed. The first one consists of parallel connections of saturation functions and the other one consists of nested saturation functions. Some free parameters and the so-called state-dependent saturation functions are introduced into these two types of control laws which can help to improve the transient performance of the closed-loop system. In the presence of input delay, with the aid of a special canonical form, two types of nonlinear control laws are also proposed to achieve global stability of the closed-loop system. Two numerical examples are given to illustrate the effectiveness of the proposed methods.

1. Introduction

Stabilization of control systems by using bounded controls is an important problem since any practical actuators can only provide bounded control signals. Thus such a kind of problems has been investigated extensively in the past several decades (Chen, Fei, & Li, 2017; Hara & Kojima, 2012; Hu, Lin, & Chen, 2002; Yang, Meng, Dimarogonas, & Johansson, 2014). One of the most important problem in this field is the global stabilization of linear systems with bounded controls. It is now clear that this problem is solvable if and only if the linear system is asymptotically null controllable with bounded controls (ANCBC), namely, it is stabilizable in the ordinary sense and all open-loop poles are located on the closed left-half plane (inside or on the unit circle in the discrete-time setting) (Sussmann, Sontag, & Yang, 1994; Yang, Sontag, & Sussmann, 1997). Moreover, as a special ANCBC system, multiple integrators with length $n \geq 3$ cannot be stabilized by bounded linear feedback (Sussmann & Yang, 1991). Thus nonlinear feedback is necessary for general ANCBC systems. In 1992, by using nested saturation functions and a state transformation, Teel firstly established a nonlinear controller for the global stabilization problem for multiple integrators by using bounded feedback (Teel, 1992). This pioneer work was later successfully extended to general ANCBC linear systems (continuous-time case in Sussmann et al., 1994 and discrete-time case in Yang et al., 1997) and even nonlinear systems (Ye, Wang, & Wang, 2007). A different nonlinear controller without state transformation to this problem was established in Kaliora and Astolfi (2004). Recently, a p-bounded feedback law was established for global stabilization of multiple integrators subject to actuator magnitude and rate constraints (Laporte, Chaillet, & Chitour, 2017).

Controllers designed by Teel's method generally suffer from rather slow convergence of the closed-loop system, especially for high-dimensional systems and/or large initial conditions. In order to improve the control performance, a number of explorations and modifications have been made by some researchers. For example, for global stabilization of multiple integrators, Marchand et al. proposed in Marchand (2003) and Marchand and Hably (2005) two types of nonlinear feedback laws consisting of nested and cascade state-dependent saturation functions, which can improve significantly the transient performance of the closed-loop system. These methods were further improved in Zhou and Duan (2009) and were extended to the discrete-time multiple integrators in Marchand, Hably, and Chemori (2007).

On the other hand, time delay is also frequently encountered in engineering and often leads to instability and/or performance degradation (Hale, 1977). Thus stabilization of linear systems with time-delays in the controls (bounded feedback) has also attracted much attention in the literature (Jia, Xu, Cui, Zhang, & Ma, 2017; Kojima, Uchida, Shimemura, & Ishijima, 1994; Li, Song, & Wu, 2018; Liu & Fridman, 2014; Mazenc, Mondie, & Niculescu, 2004; Selivanov & Fridman, 2016; Zhang, Deng, Zhang, & Sun, 2017; Zhang, Liu, Feng, & Zhang, 2013). Particularly, Teel's design by using

☆ This work was supported in part by the National Natural Science Foundation of China under Grant Number 61773140. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Emilia Fridman under the direction of Editor Ian R. Petersen.
* Corresponding author.
E-mail addresses: binzhou@hit.edu.cn (B. Zhou), 14B904038@hit.edu.cn (X. Yang).

https://doi.org/10.1016/j.automatica.2018.08.015
0005-1098/

nonlinear feedback has been extended to solve this problem. This was initialed by Mazenc et al. in Mazenc, Mondié, and Niculescu (2003), where a nested nonlinear controller was established to solve the global stabilization problem for continuous-time multiple integrators. The design in Mazenc et al. (2003) was based on a canonical form introduced by Teel in Teel (1992). However, because of the presence of delay in the control, the decoupled property in the recursive design for delay-free systems (Teel, 1992) is no longer valid, which makes the stability analysis in Mazenc et al. (2003) rather involved. Recently, in Zhou and Yang (2016) we provided some new solutions to such a problem based on some new special canonical forms, by which the decoupled property in the recursive design mentioned above still holds true, making the stability analysis quite easy and helping to provide very simple stability conditions. Teel's recursive design has also been extended to feedforward nonlinear systems with bounded and delayed controls (Mazenc, Mondié, & Francisco, 2004; Ye, 2014; Zhang, Liu, Baron, & Boukas, 2011).

In this paper, we investigate the problem of global stabilization of discrete-time multiple integrators by bounded and delayed controls. When the delay is absent in the control, we mainly deal with the problem of performance improvement, which will be achieved in two ways. On one hand, we establish two nonlinear controllers where the saturation functions are state-dependent, which helps to increase the control energy and thus to improve the control performance. On the other hand, we establish some new constraints on the corresponding parameters that are less conservative than those in the existing results (Marchand et al., 2007), which determines a larger range of parameters that can be well designed to improve the control performance. When the delay is present at the control, motivated by our study in Zhou and Yang (2016) for continuous-time multiple integrators, we solve the corresponding global stabilization problem in the discrete-time setting. This problem, to the best of our knowledge, seems not solved in the literature. Our solution is based on a special canonical form in which the delay is introduced to the state variable, helping to maintain the decoupled property in Teel's recursive design for delay-free systems. We mention that the extensions are nontrivial since, owing to the saturation, the continuous time controller cannot be discretized as it is nonlinear (Marchand et al., 2007). Moreover, the new canonical form for the considered system in this paper is different from that Zhou and Yang (2016) in the sense that it contains both the current and delayed state variables, while only delayed states were present in Zhou and Yang (2016). As a result, the linearized closed-loop system is different from that in Zhou and Yang (2016), and thus a different analysis method will be used.

The remainder of this paper is organized as follows. Problem formulation and preliminaries are given in Section 2. The design of controllers in the absence and presence of delay are respectively presented in Sections 3 and 4. Two illustrative examples are given in Section 5 to demonstrate the effectiveness of the proposed approaches and Section 6 concludes the paper.

Notation: The notation used in this paper is standard. For two integers p and q with $p \leq q$, the symbol $\mathbf{I}[p,q]$ refers to the set $\{p, p+1, \ldots, q\}$. For a positive constant ε, $\sigma_\varepsilon(x) \triangleq \varepsilon \mathrm{sign}(x)\min\{|x/\varepsilon|, 1\}$ denotes the standard saturation function. The notation $|\cdot|$ refers to both the induced matrix 2 norm and the usual Euclidean vector norm.

2. Problem formulation and preliminaries

In this paper, we consider the following nth order discrete-time multiple integrators (Marchand et al., 2007)

$$x(k+1) = A_\mathrm{d} x(k) + b_\mathrm{d} u(k), \tag{1}$$

with

$$A_\mathrm{d} = \begin{bmatrix} 1 & 1 & 0 & \cdots & 0 \\ & 1 & \ddots & \ddots & \vdots \\ & & \ddots & 1 & 0 \\ & & & 1 & 1 \\ & & & & 1 \end{bmatrix}, \quad b_\mathrm{d} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \tag{2}$$

where $x = [x_1, x_2, \ldots, x_n]^\mathrm{T} \in \mathbf{R}^n$ and $u \in \mathbf{R}$ are respectively the state and control variables. Our first task is to solve the following problem:

Problem 1. Find a state feedback control u with $|u| \leq 1$ such that the closed-loop system is globally asymptotically stable and locally exponentially stable at the origin.

When the input of system (1) is also subject to time-delay, system (1) can be written as:

$$x(k+1) = A_\mathrm{d} x(k) + b_\mathrm{d} u(k-r), \tag{3}$$

where r is a positive integer denoting the input delay. Our second task is to solve the following problem:

Problem 2. Find a state feedback control u with $|u| \leq 1$ such that the closed-loop system is globally asymptotically stable and locally exponentially stable at the origin.

Let λ_i, $i \in \mathbf{I}[1,n]$, be a set of priori given positive numbers and

$$A_0 = \begin{bmatrix} 1 & \lambda_2 & \cdots & \lambda_{n-1} & \lambda_n \\ & 1 & \ddots & \vdots & \vdots \\ & & \ddots & \lambda_{n-1} & \lambda_n \\ & & & 1 & \lambda_n \\ & & & & 1 \end{bmatrix}, \quad b_0 = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ 1 \end{bmatrix}. \tag{4}$$

By using the following invertible transformation (Marchand et al., 2007)

$$y(k) = Q_\mathrm{c}(A_0, b_0) Q_\mathrm{c}^{-1}(A_\mathrm{d}, b_\mathrm{d}) x(k), \tag{5}$$

where $Q_\mathrm{c}(A, b) = [b, Ab, \ldots, A^{n-1}b]$ is the controllability matrix of (A, b), system (1) can be transformed into the special form

$$y(k+1) = A_0 y(k) + b_0 u(k). \tag{6}$$

By virtue of the special state representation (6), Problem 1 has been investigated by Marchand et al. in Marchand et al. (2007) by using Teel's design (Teel, 1992). To ensure stability, the parameters λ_i, $i \in \mathbf{I}[1,n]$, should satisfy (Marchand et al., 2007)

$$0 < \sum_{i=1}^{k-1} \lambda_i < \lambda_k < 1, \ k \in \mathbf{I}[2,n]. \tag{7}$$

These constraints may degenerate the control performance since $1 - \lambda_i$ are the eigenvalues of the linearized closed-loop system.

In this paper, some new approaches will be presented to solve Problem 1. Compared with the results in Marchand et al. (2007), our constraints on parameters λ_i will be relaxed. Moreover, our methods can also be used to solve Problem 2 in the presence of input delay. To the best of our knowledge, this latter problem was not solved in the existing literature.

We end this section with a technical lemma, which will play an important role in establishing our main results. The proof will be given in Appendix A.1 for clarity.

Lemma 1. *Let $1 > \lambda > 0$, $\varepsilon > 0$, $\varepsilon_1 \geq 0$ and $\varepsilon_2 \geq 0$ be four given numbers. Consider the following discrete-time scalar system*

$$x(k+1) = x(k) + u(k) - \upsilon_2, \quad u(k) = -\sigma_\varepsilon(\lambda x(k) + \upsilon_1), \tag{8}$$