Automatica 97 (2018) 339–345

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

On the timed temporal logic planning of coupled multi-agent systems ☆

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ARTICLE INFO

Article history:
Received 17 January 2017
Received in revised form 2 May 2018
Accepted 28 July 2018

Keywords:
Multi-agent systems
Cooperative control
Hybrid systems
Formal verification
Timed logics
Abstractions
Discrete event systems

ABSTRACT

This paper presents a fully automated procedure for controller synthesis for multi-agent systems under coupling constraints. Each agent is modeled with dynamics consisting of two terms: the first one models the coupling constraints and the other one is an additional bounded control input. We aim to design these inputs so that each agent meets an individual high-level specification given as a Metric Interval Temporal Logic (MITL). First, a decentralized abstraction that provides a space and time discretization of the multi-agent system is designed. Second, by utilizing this abstraction and techniques from formal verification, we propose an algorithm that computes the individual runs which provably satisfy the high-level tasks. The overall approach is demonstrated in a simulation example conducted in MATLAB environment.

1. Introduction

Over the last few years, the field of control of multi-agent systems under high-level specifications has been gaining attention. In this work, we aim to additionally introduce specific time bounds into these tasks, in order to include specifications such as "Robot 1 and robot 2 should visit region A and B within 4 time units, respectively", or "Both robots 1 and 2 should periodically survey regions A_1, A_2, A_3, avoid region X and always keep the longest time between two consecutive visits to A_1 below 8 time units".

The qualitative specification language that has primarily been used to express the high-level tasks is Linear Temporal Logic (LTL) (see, e.g., Wongpiromsarn, Topcu, & Murray, 2010). There is a rich body of literature containing algorithms for verification and synthesis of multi-agent systems under high level specifications (Guo & Dimarogonas, 2015; Kantaros & Zavlanos, 2016; Saha, Ramaithitima, Kumar, Pappas, & Seshia, 2016). Controller synthesis under timed specifications has been considered in Fu and Topcu (2015), Liu and Prabhakar (2014), Raman, Donzé, Sadigh, Murray, and Seshia (2015) and Zhou, Maity, and Baras (2016). In Liu and Prabhakar (2014), the authors addressed the problem of designing high-level planners to achieve tasks for switching dynamical systems under Metric Temporal Logic (MTL) specifications and in Raman et al. (2015), the authors utilized a counterexample-guided synthesis for cyber–physical systems subject to Signal Temporal Logic (STL) specifications. In Fu and Topcu (2015), an optimal control problem for continuous-time stochastic systems subject to objectives specified in MITL was studied. In Zhou et al. (2016), the authors focused on motion planning based on the construction of an efficient timed automaton from a given MITL specification. However, all these works are restricted to single agent planning and are not extendable to multi-agent systems in a straightforward way. High-level coordination of multiple vehicles under timed specifications has been considered in Karaman and Frazzoli (2008), by solving an optimization problem over the tasks' execution time instances.

An automata-based solution for multi-agent systems was proposed in our previous work (Nikou, Tumova, & Dimarogonas, 2016), where Metric Interval Temporal Logic (MITL) formulas were introduced in order to synthesize controllers such that every agent fulfills an individual specification and the team of agents fulfills a global task. Specifically, the abstraction of each agent's dynamics was considered to be given and an upper bound of the time that each agent needs to perform a transition from one region to

☆ This work was supported by the H2020 ERC Starting Grant BUCOPHSYS, the EU H2020 Research and Innovation Programme, Sweden under GA No. 731869 (Co4Robots), the SSF COIN project, the Swedish Research Council (VR) and the Knut och Alice Wallenberg Foundation. The material in this paper was partially presented at the 2017 American Control Conference, May 24–26, 2017. This paper was recommended for publication in revised form by Associate Editor Joerg Raisch under the direction of Editor Christos G. Cassandras.

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https://doi.org/10.1016/j.automatica.2018.08.023
0005-1098/

another was assumed. Furthermore, potential coupled constraints between the agents were not taken into consideration. Motivated by this, in this work, we aim to address the aforementioned issues. We assume that the dynamics of each agent consists of two parts: the first part is a consensus type term representing the coupling between the agent and its neighbors, and the second one is an additional control input which will be exploited for high-level planning. Hereafter, we call it a free input. A decentralized abstraction procedure is provided, which leads to an individual Transition System (TS) for each agent and provides a basis for high-level planning. Additionally, this abstraction is associated to a time quantization which allows us to assign precise time durations to the transitions of each agent. Abstractions for both single and multi-agent systems can be found in Alur, Henzinger, Lafferriere, and Pappas (2000), Boskos and Dimarogonas (2015), Hussein, Ames, and Tabuada (2017), Meyer, Girard, and Witrant (2017), Zamani, Mazo, and Abate (2014) and Zamani, Pola, Mazo, and Tabuada (2012). Compositional frameworks are provided in Meyer et al. (2017) for safety specifications of discrete time systems, and Hussein et al. (2017), which is focused on feedback linearizable systems with a cascade interconnection. In addition, local invariant sets for discrete time coupled linear systems are considered in Nilsson and Ozay (2016) and are leveraged for control synthesis. The above results are therefore not applicable to the decentralized abstraction of the multi-agent control systems we consider, which evolve in continuous time and do not require a specific network interconnection.

Motivated by our previous work (Boskos & Dimarogonas, 2015), we start from the consensus dynamics of each agent and we construct a Weighted Transition System (WTS) for each agent in a decentralized manner. Each agent is assigned an individual task given in MITL formulas. We aim to design the free inputs so that each agent performs the desired individual task within specific time bounds. In particular, we provide an automatic controller synthesis method for coupled multi-agent systems under high-level tasks with timed constraints. A motivation for this framework comes from applications such as the deployment of aerial robotic teams. In particular, the consensus coupling allows the robots to stay sufficiently close to each other and maintain a connected network during the evolution of the system. Additionally, individual MITL formulas are leveraged to assign area monitoring tasks to each robot individually. The MITL formalism enables us to impose time constraints on the monitoring process. The interested reader is referred to Nikou, Boskos, Tumova, and Dimarogonas (2017) for an extended version of this paper that includes additional examples, detailed derivations and proofs.

2. Notation and preliminaries

Denote by $\mathbb{R}, \mathbb{Q}_+, \mathbb{N}$ the set of real, nonnegative rational and natural numbers including 0, respectively. Given a set S, we denote by $|S|$ its cardinality, by $S^N = S \times \cdots \times S$, its N-fold Cartesian product and by 2^S the set of all its subsets. For a subset S of $\mathbb{R}^n$, denote by $\mathrm{cl}(S)$, $\mathrm{int}(S)$ and $\partial S = \mathrm{cl}(S) \setminus \mathrm{int}(S)$ its closure, interior and boundary, respectively. The notation $\|x\|$ is used for the Euclidean norm of a vector $x \in \mathbb{R}^n$ and $\|A\| = \max\{\|Ax\| : \|x\| = 1\}$ for the induced norm of a matrix $A \in \mathbb{R}^{m\times n}$; an *undirected graph* $\mathcal{G}$ is a pair $(\mathcal{I}, \mathcal{E})$, where $\mathcal{I} = \{1, \ldots, N\}$ is a finite set of nodes, representing a team of agents, and $\mathcal{E} \subseteq \{\{i, j\} : i, j \in \mathcal{I}, i \neq j\}$, is the set of edges that model the communication capability between the neighboring agents. For each agent, its neighbors' set $\mathcal{N}(i)$ is defined as $\mathcal{N}(i) = \{j_1, \ldots, j_{N_i}\} = \{j \in \mathcal{I} : \{i, j\} \in \mathcal{E}\}$ where $N_i = |\mathcal{N}(i)|$. The Laplacian matrix $L(\mathcal{G}) \in \mathbb{R}^{N\times N}$ of the graph $\mathcal{G}$ is defined as $L(\mathcal{G}) = D(\mathcal{G})D(\mathcal{G})^\top$ where $D(\mathcal{G})$ is the $N \times |\mathcal{E}|$ incidence matrix, as it is defined in Mesbahi and Egerstedt (2010, Chapter 2). If we consider an ordering $0 = \lambda_1(\mathcal{G}) \leq \lambda_2(\mathcal{G}) \leq \cdots \leq \lambda_N(\mathcal{G}) = \lambda_{\max}(\mathcal{G})$ of the eigenvalues of $L(\mathcal{G})$ then we have that $\lambda_2(\mathcal{G}) > 0$ iff $\mathcal{G}$ is connected (Mesbahi & Egerstedt, 2010, Chapter 2). Denote by $\widetilde{x} \in \mathbb{R}^{|\mathcal{E}|n}$ the stack column vector of the vectors $x_i - x_j$, $\{i, j\} \in \mathcal{E}$ with the edges ordered as in the case of the incidence matrix $D(\mathcal{G})$.

Definition 1. A cell decomposition $S = \{S_\ell\}_{\ell\in\mathbb{I}}$ of a set $\mathcal{D} \subseteq \mathbb{R}^n$, where $\mathbb{I} \subseteq \mathbb{N}$ is a finite or countable index set, is a family of uniformly bounded convex sets $S_\ell, \ell \in \mathbb{I}$ such that $\mathrm{int}(S_\ell) \cap \mathrm{int}(S_{\hat{\ell}}) = \emptyset$ for all $\ell, \hat{\ell} \in \mathbb{I}$ with $\ell \neq \hat{\ell}$ and $\cup_{\ell\in\mathbb{I}} S_\ell = \mathcal{D}$. The interiors of the cells are non-empty.

Definition 2 (*Alur & Dill, 1994*)**.** A time sequence $\tau = \tau(0)\tau(1)\ldots$ is an infinite sequence of time values $\tau(j) \in \mathbb{T}$, with $\mathbb{T} = \mathbb{Q}_+$, satisfying the following properties: Monotonicity: $\tau(j) < \tau(j + 1)$ for all $j \geq 0$; Progress: For every $t \in \mathbb{T}$, there exists $j \geq 1$, such that $\tau(j) > t$.

Definition 3 (*Alur & Dill, 1994*)**.** An atomic proposition p is a statement that is either True ($\top$) or False ($\bot$). Let AP be a finite set of atomic propositions. A timed word w over the set AP is an infinite sequence $w^t = (w(0), \tau(0))(w(1), \tau(1))\ldots$ where $w(0)w(1)\ldots$ is an infinite word over the set 2^{AP} and $\tau(0)\tau(1)\ldots$ is a time sequence with $\tau(j) \in \mathbb{T},\ j \geq 0$.

Definition 4. A Weighted Transition System (WTS) is a tuple $(S, S_0, Act, \longrightarrow, d, AP, L)$ where S is a finite set of states; $S_0 \subseteq S$ is a set of initial states; Act is a set of actions; $\longrightarrow \subseteq S \times Act \times S$ is a transition relation; $d : \longrightarrow \to \mathbb{T}$ is a map that assigns a positive weight to each transition; AP is a finite set of atomic propositions; and $L : S \to 2^{AP}$ is a labeling function. For every $s \in S$ and $\alpha \in Act$ define $\mathrm{Post}(s, \alpha) = \{s' \in S : (s, \alpha, s') \in \longrightarrow\}$.

Definition 5. A timed run of a WTS is an infinite sequence $r^t = (r(0), \tau(0))(r(1), \tau(1))\ldots$, such that $r(0) \in S_0$, and for all $j \geq 1$, it holds that $r(j) \in S$ and $(r(j), \alpha(j), r(j + 1)) \in \longrightarrow$ for a sequence of actions $\alpha(1)\alpha(2)\ldots$ with $\alpha(j) \in Act, \forall j \geq 1$. The time stamps $\tau(j), j \geq 0$ are inductively defined as follows: (1) $\tau(0) = 0$; (2) $\tau(j + 1) = \tau(j) + d(r(j), \alpha(j), r(j + 1)),\ \forall j \geq 1$. Every timed run r^t generates a timed word $w(r^t) = (w(0), \tau(0))\,(w(1), \tau(1))\ldots$ over the set $2^{AP} \times \mathbb{T}$ where $w(j) = L(r(j)), \forall j \geq 0$ is the subset of atomic propositions that are true at state $r(j)$.

The syntax of *Metric Interval Temporal Logic (MITL)* over a set of atomic propositions AP is defined by the grammar: $\varphi := p \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \bigcirc_I \varphi \mid \Diamond_I \varphi \mid \Box_I \varphi \mid \varphi_1\, \mathcal{U}_I\, \varphi_2$, where $p \in AP$, and $\bigcirc$, $\Diamond$, $\Box$, and $\mathcal{U}$, are the next, eventually, always, and until, temporal operator, respectively; $I = [a, b] \subseteq \mathbb{T}$ where $a, b \in [0, \infty]$ with $a < b$ is a non-empty timed interval. The MITL formulas are interpreted over timed words like the ones produced by a WTS which is given in Definition 5. The semantics of MITL can be found in Nikou et al. (2017, Section 2). It has been proved that MITL is decidable in infinite words and point-wise semantics, which is the case considered here (see Alur, Feder, & Henzinger, 1996 for details).

Let $C = \{c_1, \ldots, c_{|C|}\}$ be a finite set of *clocks*. The set of *clock constraints* $\Phi(C)$ is defined by the grammar: $\phi := \top \mid \neg\phi \mid \phi_1 \wedge \phi_2 \mid c \bowtie \psi$, where $c \in C$ is a clock, $\psi \in \mathbb{T}$ is a clock constant and $\bowtie\ \in \{<, >, \geq, \leq, =\}$. A clock *valuation* is a function $\nu : C \to \mathbb{T}$ that assigns a value to each clock.

Definition 6 (*Alur & Dill, 1994*; *Bouyer, 2009*; *Tripakis, 2009*)**.** A *Timed Büchi Automaton* is a tuple $\mathcal{A} = (Q, Q^{\mathrm{init}}, C, Inv, E, F, AP, \mathcal{L})$ where Q is a finite set of locations; $Q^{\mathrm{init}} \subseteq Q$ is the set of initial locations; C is a finite set of clocks; $Inv : Q \to \Phi(C)$ is the invariant; $E \subseteq Q \times \Phi(C) \times 2^C \times Q$ gives the set of edges of the form $e = (q, \gamma, R, q')$, where q, q' are the source and target states, γ is the guard of edge e and R is a set of clocks to be reset upon