Automatica 97 (2018) 404–413

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

A sliding mode approach to stabilization of nonlinear Markovian jump singularly perturbed systems ☆

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ARTICLE INFO

Article history:
Received 16 January 2017
Received in revised form 5 January 2018
Accepted 2 February 2018
Available online 13 September 2018

Keywords:
Sliding mode control (SMC)
Markovian jump systems
Singularly perturbed systems (SPS)

ABSTRACT

This paper presents a sliding mode control (SMC) method for a class of nonlinear Markovian jump singularly perturbed systems (MJSPSs). The system is subject to parameter uncertainties and partly unknown transition probabilities. To fully employ the model characteristics of such a hybrid system, a novel integral-type switching function is firstly designed. By adopting the ε-dependent stochastic Lyapunov function method, sufficient conditions are presented to ensure the mean-square asymptotic stability of the sliding mode dynamics. A mode-dependent fuzzy SMC law is then synthesized to induce and maintain the sliding motion despite partly unknown transition probabilities and parameter uncertainties. Finally, the developed method is applied to stabilize a modified series DC motor system.

1. Introduction

It is well known that many processes inherently have a two-scale dynamic, which can be exactly modeled by singularly perturbed systems (SPSs) (Shi & Dragan, 1999). The key feature of an SPS lies in that there is a small parasitic parameter multiplying the time derivatives of a part of the states in the model, which makes the system stiff and unwieldy. Take the power system as an example, the small parasitic parameter may represent the machine reactance or transient in the voltage regulator. Some results on analysis and synthesis for SPSs (Chen, Wang, Wei, & Lu, 2014; Fridman, 2002; Karimi, Yazdanpanah, & Khorasani, 2006; Vecchio & Slotine, 2013; Xu & Feng, 2009; Yang, Sun, & Ma, 2013) have been reported in the recent years. It is worth mentioning that the aforementioned results are mainly confined to linear SPSs. For nonlinear SPSs, especially for highly complex nonlinear SPSs, it is difficult to decompose them into slow and fast subsystems, and the analysis is more complicated than the linear case. Recently, T–S fuzzy model approach has been proposed to deal with the complex nonlinear systems. In view of its great success, considerable attention has been devoted to controller design for fuzzy SPSs (Yang & Dong, 2008; Yang & Zhang, 2009). With T–S fuzzy models, the nonlinear SPS can be approximated by the weighted sum of a series of linear subsystem. Thus, the relatively mature linear SPS theory can be utilized (Liu, Wu, Zhou, & Lam, 2015).

In practice, the SPSs may experience abrupt changes in their structures and parameters. The abrupt changes may come from the parameters shifting or component failures, which may severely degrade the system performance and even lead to instability (Karimi, 2011; Li, Gao, Shi, & Zhao, 2014). The hybrid systems, which involve both time-evolving and event-driven mechanisms, may be a natural representation of above problem. A special type of hybrid systems is Markovian jump singularly perturbed systems (MJSPSs). The control of linear or fuzzy MJSPSs has been an important research topic and a number of approaches have been available in the literature for controlling linear or fuzzy MJSPSs (Liu, Sun, & Sun, 2004; Wang, Huang, Zhang, & Yang, 2014). It is worth mentioning that the existing results on linear or fuzzy MJSPSs are based on a common assumption that the transition probabilities of the underlying Markov chain are completely known. Although norm bounded uncertainties in transition probabilities are considered in Wang et al. (2014), the "nominal" terms of transition probabilities still need to be known in advance. As is illustrated in Zhang and Boukas (2009) and Zhang and Lam (2010), however, incomplete transition probabilities often exist in practical applications since it is time-consuming or too costly to obtain the enough samples of the transitions.

☆ This work was partially supported by the National Natural Science Foundation of China (61622302, 61703275, U1611262, 61573070, 61425009), the Fundamental Research Funds for the Central Universities (2017FZA5010), the China Postdoctoral Science Foundation (2016M590360) and the Department of Education of Liaoning Province (LZ2017001). The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Michael V. Basin under the direction of Editor Ian R. Petersen.

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https://doi.org/10.1016/j.automatica.2018.03.066
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On the other hand, as a fruitful research frontier of the control community, the sliding mode control (SMC) has a number of attractive advantages, such as fast response and complete compensation of matched uncertainties and disturbances when the system is in the sliding phase. The SMC problems have been well addressed for a variety of complex systems, such as, stochastic systems, time-delay systems, Markovian jump systems, and descriptor systems (Basin, Ferreira, & Fridman, 2006; Basin, Panathula, Shtessel, & Ramirez, 2016; Basin & Ramirez, 2012; Chen, Niu, & Zou, 2013a, b; Kao, Xie, Wang, & Karimi, 2015; Li, Shi, Yao, & Wu, 2016; Niu, Daniel, & Lam, 2005; Shi, Xia, Liu, & Rees, 2006; Song, Niu, & Zou, 2015; Wang, Shen, Karimi, & Duan, 2017; Wu, Shi, & Gao, 2010; Wu, Su, & Shi, 2012). Meanwhile, several SMC methodologies have been extended to accommodate SPSs (Gao, Sun, & Lu, 2011; Lin, 2014; Nagarale & Patre, 2014). However, it should be pointed out that only the linear MJSPSs with complete knowledge of transition rate were considered in the literature. Although it is of more practical importance, the corresponding SMC problem for T–S fuzzy-model-based nonlinear MJSPSs with partly unknown transition probabilities has not yet been adequately investigated probably due to the difficulties in accommodating the model characteristics of such hybrid systems. The nonlinear characteristics of T–S fuzzy models and the singular perturbation structure of SPSs increase substantial challenges to the SMC design, not to mention the difficulties brought from the stochastic transition of system parameters and partly unknown transition probabilities. Correspondingly, the following three key questions need to be addressed during the SMC design:

Q1. The switching function proposed in existing SMC results is not workable for fuzzy MJSPSs due to the model complexity. In this case, how to design a suitable switching function to fully accommodate the model characteristics of such systems?

Q2. The existing methods are only applicable for linear or fuzzy MJSPSs with completely known transition probabilities. When sliding modes take place, how to obtain stochastic stabilization conditions for fuzzy MJSPSs in the presence of partly unknown transition probabilities?

Q3. How to analyze an SMC to induce and keep a sliding motion despite parameter uncertainties, stochastic switching of systems modes and partly unknown transition probabilities?

Summarizing the above discussions, in this paper, we aim to address the SMC problem for a class of fuzzy MJSPSs subject to parameter uncertainties and partly unknown transition probabilities. Firstly, a novel integral-type switching function is designed to fully capture the model characteristics. Secondly, sufficient conditions are presented to ensure the mean-square asymptotic stability of the sliding mode dynamics despite parameter uncertainties and partly unknown transition probabilities. Then, a fuzzy SMC law is synthesized to guarantee the reaching condition despite parameter uncertainties and partly unknown transition probabilities. Finally, a practical example on modified series DC motor is employed to illustrate the applicability of developed approach.

Notations: The matrix transposition is represented by superscript T. Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ represent, respectively, the n-dimensional Euclidean space and the set of all $m \times n$ real matrices. $\mathcal{M}^{\perp}(x) \in \mathbb{R}^{n\times(n-m)}$ represents the matrix with independent columns that span the null space of $\mathcal{M}(x) \in \mathbb{R}^{n\times m}$. span$\{\mathcal{M}_i\}$ denotes the set of all linear combinations composed of vectors $\mathcal{M}_i$. Ellipsis "$\cdots$" represents the omitted matrices and number. Notation "*" denotes the term induced by symmetry. The superscript "+" indicates the left inverse of a matrix. $diag\{\cdots\}$ denotes the block diagonal matrix, and I represents the identity matrix. $(\Omega, \mathcal{F}, \mathcal{P})$ denotes a probability space, where Ω, $\mathcal{F}$ and $\mathcal{P}$ represent, respectively, the sample space, the σ-algebra of subsets of the sample space, and the probability measure.

2. System description and preliminaries

Let $\{r_t, t \geq 0\}$ be a continuous-time Markov process with a right continuous trajectory which takes values in a finite set $\mathcal{S} = \{1, 2, \ldots, s\}$ with transition rate matrix $\Pi = (\pi_{pq})_{s\times s}$ given by

$$\mathcal{P}\{r_{t+\Delta} = q | r_t = p\} = \begin{cases} \pi_{pq}\Delta + o(\Delta) & \text{if } q \neq p \\ 1 + \pi_{pq}\Delta + o(\Delta) & \text{if } q = p \end{cases} \tag{1}$$

where $\Delta > 0$ and $\lim_{\Delta\to 0} o(\Delta)/\Delta = 0$; $\pi_{pq} > 0$, $q \neq p$ and $\pi_{pq} = -\sum_{q\neq p}\pi_{pq}$ for each $p \in \mathcal{S}$. It is assumed that the transition rates or probabilities of the jumping process are only partly accessed, which means some elements in matrix Π are unknown. For instance, for a system with 4 operation modes, the transition rate matrix Π may be depicted as

$$\Pi = \begin{bmatrix} \pi_{11} & \pi_{12} & ? & \pi_{14} \\ ? & \pi_{12} & ? & \pi_{24} \\ \pi_{31} & ? & ? & \pi_{34} \\ ? & \pi_{42} & \pi_{43} & ? \end{bmatrix}$$

with "?" denoting the inaccessible elements. For notational clarity, $\forall p \in \mathcal{S}$, we denote $\mathcal{S} = \mathcal{S}_k^p \bigcup \mathcal{S}_{uk}^p \bigcup \mathcal{S}_{k,p}^p \bigcup \mathcal{S}_{uk,p}^p$ with $\mathcal{S}_k^p \triangleq \{q : \pi_{pq} \text{ is known}, \ q \neq p\}$, $\mathcal{S}_{k,p}^p \triangleq \{p : \pi_{pp} \text{ is known}\}$, $\mathcal{S}_{uk}^p \triangleq \{q : \pi_{pq} \text{ is unknown}, \ q \neq p\}$, $\mathcal{S}_{uk,p}^p \triangleq \{p : \pi_{pp} \text{ is unknown}\}$.

Moreover, if $\mathcal{S}_k^p \neq \emptyset$, it is further represented as $\mathcal{S}_k^p = \{\mathcal{K}_1^p, \mathcal{K}_2^p, \ldots, \mathcal{K}_{m_p}^p\}$, $\forall 1 \leq m_p \leq s$ where $\mathcal{K}_{m_p}^p \in \mathbb{N}^+$ represents the m_pth known element with the index $\mathcal{K}_{m_p}^p$ in the pth row of the transition rate matrix Π. Also, we denote $\widehat{\pi}_k^p = 1 + \sum_{q\in\mathcal{S}_k^p}\pi_{pq}$.

Fix a probability space $(\Omega, \mathcal{F}, \mathcal{P})$, and consider the following uncertain fuzzy MJSPS, which can be exactly described by the following fuzzy rules:

Model Rule i: IF $z_1(x(t))$ is η_{i1}, $z_2(x(t))$ is η_{i2}, $\cdots$, $z_l(x(t))$ is η_{il}, THEN

$$\begin{aligned} \dot{x}_1(t) &= \big(A_{11i}(r_t) + \Delta A_{11i}(r_t, t)\big)x_1(t) + \big(A_{12i}(r_t) \\ &\quad + \Delta A_{12i}(r_t, t)\big)x_2(t) + B_{1i}(r_t)\Big(u(t) + f\big(x(t), r_t\big)\Big) \\ \varepsilon\dot{x}_2(t) &= \big(A_{21i}(r_t) + \Delta A_{21i}(r_t, t)\big)x_1(t) + \big(A_{22i}(r_t) \\ &\quad + \Delta A_{22i}(r_t, t)\big)x_2(t) + B_{2i}(r_t)\Big(u(t) + f\big(x(t), r_t\big)\Big) \\ &\quad i = 1, 2, \ldots, r \end{aligned} \tag{2}$$

where $\eta_{i1}, \eta_{i2}, \ldots, \eta_{il}$ are the fuzzy sets, r is the number of fuzzy rules, $z_1(x(t)), z_2(x(t)), \ldots, z_l(x(t))$ are the premise variables, and $\varepsilon > 0$ is the singular perturbation parameter. $x_1(t) \in \mathbb{R}^{n_1}$ and $x_2(t) \in \mathbb{R}^{n_2}$ are the state vectors, $u(t) \in \mathbb{R}^m$ is the control input. $A_{11i}(r_t) \in \mathbb{R}^{n_1\times n_1}$, $A_{12i}(r_t) \in \mathbb{R}^{n_1\times n_2}$, $A_{21i}(r_t) \in \mathbb{R}^{n_2\times n_1}$, $A_{22i}(r_t) \in \mathbb{R}^{n_2\times n_2}$, $B_{1i}(r_t) \in \mathbb{R}^{n_1\times m}$, $B_{2i}(r_t) \in \mathbb{R}^{n_2\times m}$ are known constant matrices. $\Delta A_{11i}(r_t, t) \in \mathbb{R}^{n_1\times n_1}$, $\Delta A_{12i}(r_t, t) \in \mathbb{R}^{n_1\times n_2}$, $\Delta A_{21i}(r_t, t) \in \mathbb{R}^{n_2\times n_1}$, $\Delta A_{22i}(r_t, t) \in \mathbb{R}^{n_2\times n_2}$ are parameter uncertainties and $f(x(t), r_t) \in \mathbb{R}^m$ represents unknown nonlinear function.

Define

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, E(\varepsilon) = \begin{bmatrix} I_{n_1} & 0_{n_1\times n_2} \\ 0_{n_2\times n_1} & \varepsilon I_{n_2} \end{bmatrix}, \ n = n_1 + n_2,$$

$$\Delta A_i(r_t, t) = \begin{bmatrix} \Delta A_{11i}(r_t, t) & \Delta A_{12i}(r_t, t) \\ \Delta A_{21i}(r_t, t) & \Delta A_{22i}(r_t, t) \end{bmatrix},$$

$$A_i(r_t) = \begin{bmatrix} A_{11i}(r_t) & A_{12i}(r_t) \\ A_{21i}(r_t) & A_{22i}(r_t) \end{bmatrix}, B_i(r_t) = \begin{bmatrix} B_{1i}(r_t) \\ B_{2i}(r_t) \end{bmatrix}.$$

For the sake of simplicity, in the sequel, for each possible $r_t = p \in \mathcal{S}$, we write $A_i(r_t) \triangleq A_{p,i}$, $\Delta A_i(r_t, t) \triangleq \Delta A_{p,i}$, $B_i(r_t) \triangleq B_{p,i}$, and $f(x(t), r_t) \triangleq f_p(x(t), t)$. By using a standard fuzzy inference method,