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Research paper

Dynamics of front-like water evaporation phase transition interfaces

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ABSTRACT

We study global dynamics of phase transition evaporation interfaces in the form of traveling fronts in horizontally extended domains of porous layers where a water located over a vapor. These interfaces appear, for example, as asymptotics of shapes of localized perturbations of the unstable plane water evaporation surface caused by long-wave instability of vertical flows in the non-wettable porous domains. Properties of traveling fronts are analyzed analytically and numerically. The asymptotic behavior of perturbations are described analytically using propagation features of traveling fronts obeying a model diffusion equation derived recently for a weakly nonlinear narrow waveband near the threshold of instability. In context of this problem the fronts are unstable though nonlinear interplay makes possible formation of stable wave configurations. The paper is devoted to comparison of the known results of front dynamics for the model diffusion equation, when two phase transition interfaces are close, and their dynamics in general situation when both interfaces are sufficiently far from each other.

1. Introduction

We treat a particular class of flows in a porous medium that are subjected to a transition to instability via a zero critical wave number. These are flows in extended horizontal domains of porous media with a phase transition occurring on some interface within the flow region. As an example, we may consider a model describing filtration processes in natural massifs, having contact with mines, tunnels and other constructions. The functioning of such engineering systems is accompanied by heat and mass exchange between the construction and surrounding rock [1]. Artificial ventilation makes it possible to keep the micro-climate, necessary for exploitation. Ventilation is accompanied by evaporation from a ceiling of the construction while the ground water moves downwards under the action of gravity or pressure in the water horizon. The water can enter the underground construction either in liquid or vapor states. If the surrounding rock has relatively low permeability it is natural to assume that the underground water moving towards the ceiling of the construction evaporates somewhere in a porous space and diffuses into the underground construction as a vapor. In this case a region saturated with a blend of vapor and air arises between the free space and the water saturated region.

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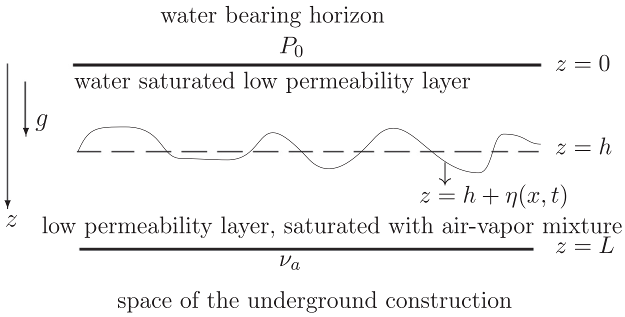

Fig. 1. Schematic of the system considered; see text for definitions.

For the transition with the most unstable mode of infinite length (zero wave number) as the perturbations grow monotonically, the evolution of the narrow band of weakly nonlinear modes near the instability threshold is described by a diffusion Kolmogorov–Petrovskii–Piskounov–Fisher (KPP) equation with a non-degenerate quadratic nonlinearity in the generic case (see [2]). This kind of equation was first considered in [3] to describe an increase in the amount of substance as applied to a biological problem. It has some interesting properties, which have been discussed in numerous publications (see, e.g., [4,5] and references therein).

It has been shown that, if the porous medium is non-wettable, there can be two or none stationary plane phase transition interfaces. At the zero wave number, the stability margin is reached simultaneously with the vanishing of the solution to the stationary problem [2]. As was mentioned the dynamics of the system in question near the instability threshold is described by the KPP equation. In this case both phase transition interfaces are located close to each other.

The model studied below provides the possibility of detecting fundamental physical effects of an evolution of perturbations of the vertical base flow between the existing phase transition interface close to and far away from the instability threshold. Conceptually, this work continues the study performed in [6]. More specifically, in [6] we treat the evolution of localized finite perturbations of the basic flow. In this case, numerical methods are required and, along with the study of physical effects, we can determine the limits of applicability of the fundamental results obtained in [2] for the case of finite amplitude localized perturbations, i.e. for problems of practical importance.

Perturbation of the phase transition interfaces of the base flow separating a fluid and vapor, plays a particular role because they are solutions of the base system of equations and, hence, any perturbation of them has to tend asymptotically to these surfaces. Dynamics of surfaces initially separating the fluid and vapor was investigated in [7]. It was shown that they tend either to stable base interface, or the highest boundary of the reservoir, depending on their initial location.

The present paper deals with a description of dynamics of fronts in the full model, which may appear, for example, due to long-time evolution of a localized perturbation of the upper unstable front. In this case, the perturbation remaining localized has increasing support. Its lateral boundaries after some time propagate as traveling fronts. In case when the stable and unstable interfaces are close and the KPP equation describes such a configuration these traveling fronts are corresponding solutions of this equation. We discuss stability of these solutions and also possibility of existence of similar solutions when the phase transition interfaces are not close (the system is far from the threshold of instability).

The paper is organized as follows. In Section 2 we give the formulation of the problem, in Section 3 we recall the dynamics of initial horizontal interfaces and also the basic KPP equation describing the dynamics of the system near the threshold of instability. In Section 4 we make a comparative analysis of the known propagation properties of front solutions of the model KPP equation describing the dynamics of the system near the threshold of instability and their properties in general case, when the phase transition interfaces are not close. We show that KPP fronts well describe the situation in general case. Section 5 is devoted to discussion of stability properties in the case of close phase transition interfaces when the fronts have the oscillatory structure (which is possible in our model). In Section 6 we give our conclusions.

2. Dynamics of the system

Let the high permeability water horizon with the water pressure P_0, bounded from above by the plane $z = 0$, be located over the ceiling $z = L$ (the z-axis is directed downwards). The rock in a layer $0 < z < L$ has a low permeability and at the surface $z = L$ it is streamlined by air of humidity ν_a which is smaller than the humidity of saturation, i.e. the partial pressure in the air is smaller than the pressure of saturation of the vapor in the air at a given value of temperature T. In this case the low permeability porous media $0 < z < L$ contains the water layer $0 < z < h$ and the layer $h < z < L$, saturated by a mixture of the air and water vapor (Fig. 1) and (is adjacent to the space of the underground construction $z > L$ (see Fig. 1). The x-axis is directed horizontally.