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Over-critical load analysis for residual stresses and displacement around fastener-holes based on the decohesive failure mechanism and non-linear Mises yield criterion

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ABSTRACT

Analytical calculations of residual stresses and displacement in the case of elevated loads applied to common engineering structures require particular attention. This is related to various types of singularities such as material and geometrical discontinuities or numerical errors, particularly, for elastic/perfectly-plastic material. In the present study, some important aspects leading to such singularities are discussed within the framework of small strains and plane-stress state. Structures such as double-shear bolted connections, shrink-fits, open fastener-holes and pressurized rigid containers are analyzed based on the classical problem of an annular plate with a central hole subject to uniformly distributed inner pressure. At the outer edge, the plate is either stress-free or embedded into a rigid container. Comparison of the results with related papers is provided.

1. Introduction

Cold expansion process is widely used in industry to introduce beneficial compressive residual stresses around fastener-holes in compound structures. These holes can be interference-fitted with bolts (pins) or left alone while fatigue remote load is applied at the outer edge of the plate (Badkhor et al., 2017; Pinho et al., 2005). Alternatively, such a compound plate may be inserted into a rigid container (Aleksandrova, 2015). In any case, compressive circumferential residual stresses created around the hole improve significantly structural performance by delaying propagation of cracks and deterioration of the plate components due to fatigue load.

Nowadays, cold expansion process is mostly studied for tubes and cylinders (Vullo, 2014) as a part of autofrettage technology, and much less attention is paid to plane compound structures. However, plane problems in engineering plasticity should be treated separately due to inherent singularities and modeling specifications (Alexandrov, 2015; Chen, 1973).

The origin of the plane-stress hole expansion problem with analytical solution for stresses is dating back to Nadai (1931) followed by his fundamental well-known book (Nadai, 1950) which includes the famous solution of Mises (1949) for an annular finite ring made of elastic/perfectly-plastic material. Later on, the complete analytical stress-strain solution to this problem was obtained by Chen (1973) based also on the same elastic/perfectly-plastic model with the main objective to compare deformation and flow theories of plasticity. At the same time, a similar problem (of an infinite plate with a rigid inclusion) was considered by Szuwalski and Zyczkowski (1973) pointing out the significance of the elastic/perfectly-plastic material in engineering calculations. One of the main contributions of that work was the development of the decohesive carrying capacity criterion. Using the Mises yield condition for plastic flow, it was shown (Szuwalski and Zyczkowski, 1973) that the deformation process cannot be continued up to the material limit load but rather terminates due to infinite increase in radial strains at the border of the rigid insert and main plate. Since then, the concept of decohesive failure mechanism is considered as a paradox in perfect plasticity that was observed in some other related engineering studies (Debski and Zyczkowski, 2000; Latas and Zyczkowski, 2000; Szuwalski, 2000).

Another paradox of the elastic/perfectly-plastic material is revealed when combined with the Tresca yield condition. For the first time, considering a rotating solid disk problem, Gamer (1983) demonstrated that Tresca yield criterion failed to predict continuous stress/displacement fields. Later on, the same phenomenon was discovered for a stationary shrink-fit problem (Gamer, 1986), however, only in the case of elevated loads.

Without doubt this phenomenon cannot be ignored since, besides cold expansion process, shrink-fit is another industrial technique used frequently to introduce residual stresses in a structure, and, hence, must be carefully modeled. In this process, a slightly oversized inner ring or

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solid disk is inserted into the outer main plate as in bolted connections (Chakherlou et al., 2010; Moze and Beg, 2014), or a slightly undersized outer thin ring is tightly fitted with the inner main plate as in pressurized cylinders (Bhatnagar, 2013).

In terms of mathematical modeling, cold expansion and shrink-fit processes are different with regard to the choice of loading parameter. For cold expansion process, the loading parameter is a pressure (applied around the hole), and, for shrink-fit process, it is the interference between the matching parts. However, these parameters are not independent and must satisfy certain rules of consistency. For the first time, it was investigated by Gamer (1986), in fact, resulting in aforementioned paradox in perfect plasticity. Specifically, it was proved that for small strains, plane-stress state and elastic/perfectly-plastic material combined with the Tresca yield criterion no solution exists for shrink-fit problem in the case of high interference level because of discontinuity either in circumferential stress or radial displacement at a newly emerged plastic boundary or at the hole edge of the main plate, respectively. To avoid these discontinuities, a strain-hardening material model was proposed instead of perfect plasticity (Gamer, 1986). Further investigation on this material model paradox was conducted year later by Durban (1987) who also used the Tresca yield criterion and a strain-hardening material to get a continuous solution within the small strain theory. It was specifically outlined that for a strain-hardening material the circumferential stress is continuous but stays non-negative. Recently, Lai and He (2012) considered the possibility of adopting the Gamer's approach with an interference as a loading parameter directly to the solution of a cold-expansion problem. However, both the model of elastic/perfectly-plastic material and the Tresca yield criterion were chosen as a part of theoretical study. An explicit equation was derived to control one-to-one correspondence between the residual displacement ratio and working pressure at the hole edge. Nevertheless, some experimental values of interferences were too large to be treated by the theoretical model proposed. It might be the reason why some theoretical results did not agree well with experimental observations in terms of elastic-plastic boundary and residual stress distributions.

Due to importance of the singularity issues, recently Masri et al. (2010) have examined in detail another (closely related) controversy between two earlier solutions of Taylor/Hill and Bethe, both based on large strains, plane-stress state and incompressible, perfectly-plastic material. The former solution, in fact, predicted a jump to a negative circumferential stress when this stress vanishes while, conform the latter one, the circumferential stress remains zero. On the other hand, it was then shown by Masri et al. (2010) that the Taylor/Hill's solution is closer to the one based on the Mises yield criterion combined with deformation theory of plasticity, and the Bethe's solution is a good approximation of the one based on the Tresca yield criterion combined with its associated flow rule. Importantly, it was also observed, for the first time (Masri et al., 2010), but not explicitly stated, that, aside from the yield criterion, other assumptions, such as small/large strains, perfectly-plastic or strain-hardening material, influence directly occurrence of singularities in analytical solutions.

To this end, new analytical study to a similar problem of shrink-fit design (Alexandrov et al., 2015) was carried out for the Mises yield criterion with its associated flow rule, small strains, plane-stress state and elastic/perfectly-plastic material. Contrary to (Lai and He, 2012), the approach was based on the cold expansion formulation which then was adopted for a shrink-fit design. Meanwhile, the employment of the flow rule theory significantly complicated its practical engineering usage.

So, the main objective and novelty of this paper is to further investigate still existing in the literature paradox in plane perfect plasticity by showing that approach based of the Mises yield criterion with the Hencky deformation theory, small strains and elastic/perfectly-plastic material leads to continuous stress/displacement fields for elevated loads both in cold expansion and shrink-fit problems including plates embedded into a rigid container. For the first time, it is explicitly stated that, in order to preserve continuous stress/displacement fields for elevated loads, the Mises yield criterion should be used with two main assumptions, namely, small or large strains and elastic/perfectly-plastic material. As a novel approach to structural safety, it is also suggested to consider the decohesive carrying capacity criterion in order to validate the admissible range of pressures and interferences leading to reliable serviceability of the structure. Some other new aspects of the paper are: a) identification of the class of engineering materials for which both the Tresca and Mises yield criteria can be used efficiently for modeling both cold expansion and shrink-fit processes including plates embedded into a rigid container; b) identification of maximum loading parameter for which elastic unloading is valid in residual stress evaluations; c) classification of interference-fit problems for engineering convenience such as those with elastic permanent insert into the main plate, expansion of a hole by a rigid mandrel during the loading process, and permanent residual displacement at the hole edge after the pressure release.

In the present study, to exclude the effect of multiple parameters on residual stresses and displacement, only elastic/perfectly-plastic material is considered while combined with an adequate yield criterion. In spite of the fact that the merits of the Mises and Tresca conditions have been discussed previously in numerous papers, for example, the classical work by Rees (1999) for rotating disks, Pinho et al. (2005) for pressurized plates, Masri et al. (2010) for large strains, and there exists a well-known generalized yield plastic potential with Mises and Tresca solids recovered as a special case (Cohen et al., 2009), it is still important for practical engineering applications to consider them separately in order to derive simple reliable algorithm and check for singularities. In complete final modeling, however, the other key parameters such as the Baushinger effect including re-yielding and strain-hardening index should be also considered which is usually provided by three-dimensional numerical analyses (Chakherlou et al., 2010; Perl and Steiner, 2018). Excellent review on sensitivity of plane-stress and plane-strain solutions to hardening is provided by Masri et al. (2010). Meanwhile, the plane-strain approach in plasticity differs essentially from the plane-stress one (Chen, 1973).

2. Continuous stress fields

Consider a thin annular plate of elastic/perfectly-plastic material of yield stress in tension Y (MPa), Young modulus E (GPa), Poisson ratio ν, outer radius b (m), inner radius a (m) subjected to gradually increasing radial pressure $\hat{p}$ around its inner edge. The pressure is sufficient enough to bring the material into a partially plastic state dividing it into inner plastic and outer elastic zones (Fig. 1).

The inner plastic zone spreads from the center of the plate and is bounded by an unknown elastic-plastic boundary c while the outer zone of the plate remains elastic. To find stress/displacement distributions, the two zones – plastic and elastic – should be treated separately but in such a way that the condition of continuity for stresses - $\hat{\sigma}_{rr}$ (radial), $\hat{\sigma}_{\theta\theta}$(circumferential) - and radial displacement $\hat{u}$ meats at the elastic-plastic border. In addition, there are two boundary conditions for a classical plate/hole problem: $\hat{\sigma}_{rr} = -\hat{p}$ at $r = a$ and $\hat{\sigma}_{rr} = 0$ at $r = b$ (this is so called the classical plate/hole problem valid for fastener-hole structures or double-shear bolted connections) where r denotes the radial coordinate in cylindrical coordinate system $r\theta z$. Meanwhile, the analysis presented here may be extended to slightly modified boundary conditions: $\hat{\sigma}_{rr} = -\hat{p}$ at $r = a$ and $\hat{u} = 0$ at $r = b$ (this problem is referred to as pressurized rigid containers).

Within the inner plastic zone, material obeys equilibrium equation and Mises yield criterion

$$d(r\hat{\sigma}_{rr})/dr - \hat{\sigma}_{\theta\theta} = 0 \tag{1}$$

$$\hat{\sigma}_{rr}^2 + \hat{\sigma}_{\theta\theta}^2 - \hat{\sigma}_{rr}\hat{\sigma}_{\theta\theta} = Y^2 \tag{2}$$

Introducing dimensionless notations $\sigma_{rr} = \hat{\sigma}_{rr}/Y$, $\sigma_{\theta\theta} = \hat{\sigma}_{\theta\theta}/Y$,