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Saturated impulse for fully clamped square plates under blast loading

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ABSTRACT

Saturated impulse refers to the critical value, beyond which the deflection of a beam or plate under pulse loading will no longer increase with further applied load. The present paper investigates the saturated impulse of fully clamped square plates subjected to a blast loading that is assumed to be a uniformly distributed Linearly Rising Exponentially Decaying (LRED) pressure pulse. Considering both bending moment and membrane force, the transient dynamic plastic response is predicted by a rigid, perfectly plastic (R-PP) model characterized by travelling plastic hinge lines and time-dependent velocity field. The saturated duration, saturated impulse, saturated deflection, as well as the evolution of deformation mechanism are explored. By comparing three characteristic durations, i.e. the travelling duration of the plastic hinge lines, saturated duration and loading duration, the dynamic responses of a square plate are classified into three types such that a regime map on the loading parameters' plane is constructed accordingly. The correlation is made between the results obtained from the elastic-plastic numerical simulation and the R-PP theoretical approximation. To facilitate engineering designs, a method of replacing the LRED pressure pulse by an equivalent rectangular pressure pulse is proposed to predict the maximum deflection of fully clamped square plates.

1. Introduction

Explosion is a major source of threat to engineering structures, which likely causes large deflection and failure of the structures. Many studies have been reported on large deflection dynamic plastic response of the metal plates in relation to their engineering applications. However, most published theoretical approaches for predicting the dynamic plastic responses of plates/beams are based on a time-independent modal velocity field, in which the transient stage in the response history is approximately considered or even totally neglected to reduce the difficulties in the mathematical treatment [1–6]. Jones [7] explored the transient dynamic response of a simply supported beam loaded impulsively, by assuming that the motion of the travelling plastic hinges is similar to that in the small deflection model, in which only the bending moment is considered. Following this study, Zhao et al. [8] investigated the dynamic response of the same structure loaded by a rectangular pressure pulse with the same assumption. Although this assumption is helpful to the derivation of the explicit expression, it somehow lacks theoretical justification. Yu and Chen [9] proposed a theoretical procedure of tracing the large deflection dynamic response of rectangular plates. Their method is capable of incorporating the effect of the membrane forces into the transient stage of the dynamic response of the plates. Zhu [10] investigated the transient deformation modes of square plates under explosive loading, with both experimental and numerical approaches. In his study, the elastic-plastic numerical predictions showed good agreement with the experimental results in terms of both the permanent deflection and transient deformation profiles.

In general, an intensive dynamic loading (e.g. explosive loading) imposed on plates is characterized merely by the maximum (i.e. peak) load and total impulse of the loading pulse. On the other hand, many studies have revealed that the load-carrying capacity of a plate is greatly enhanced when subjected to intense transverse dynamic loading due to the membrane forces induced by the large deflection. If the applied load is a pressure pulse with a sufficiently long duration such as a rectangular or Linearly Rising Exponentially Decaying (LRED) pressure pulse, the transverse displacement of the plate would cease when the pulse duration reaches to a certain magnitude; namely, the saturation phenomenon (i.e. the saturated deflection, saturated impulse and saturated duration) will take place. Thus, a series of studies on saturated impulse (see some key references below) have indicated that the structural design based on the total impulse of the applied pulse may be misleading, since only part of the impulse (i.e. the saturated impulse) is responsible for the permanent deformation of structures.

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Notation

E	Young's modulus
H	plate thickness
I	impulse
I_e	impulse of effective rectangular pressure pulse
$\bar{I}$	dimensionless impulse
$2L$	side length of square plate
p_0	peak load of pressure pulse
p_e	amplitude of effective rectangular pressure pulse
p_y	static collapse pressure
t	time
t_0	rising duration of linearly decaying pressure pulse, or loading duration of rectangular pressure pulse
t_1	travelling duration of plastic hinges
t_f	time instant when plastic deformation ends
t_y	time instant when plastic deformation begins
$2t_{mean}$	loading duration of effective rectangular pressure pulse
W_0	deflection at the plate centre
ξL	horizontal distance between the edge of the rigid zone ① and the side of the plate
θ	a parameter with time unit (s)
λ	dimensionless peak load of a LRED pulse
λ_0	dimensionless initial load of a LRED pulse
λ_e	dimensionless equivalent amplitude of rectangular pressure pulse
μ	mass per unit area of plate
ρ	density of material
σ_Y	yield stress
ϕ	rotation angles of rigid zone ②.
ω	characteristic frequency
$()^{sat}$	saturated value
$()_f$	final value
$()_m$	maximum value

Zhao et al. [8] first illustrated the saturated impulse of a beam undergoing large deflection under a moderate rectangular pressure pulse, and provided a convincing explanation to the saturation phenomenon. The same authors [11] further extended this concept to circular plates, square plates and cylindrical shells under similar pulse loadings. Since the rigid-plastic idealization is adopted in their modelling, the above investigations on saturated impulse only identify the maximum deformation of a structure. Zhu and Yu [12] further developed the concept of the saturated impulse, by investigating both the maximum deformation and final deformation based on the elastic-plastic analysis of a clamped square plate. Recently, our group has carried out more studies to investigate the effects of various parameters on the saturated phenomenon of plates, e.g., scaling ratio of the square plate [13], strain rate and strain hardening of the material [14], the aspect ratio and boundary conditions of the rectangular plate [15], Apart from rectangular pulse loading, the saturated impulse of fully clamped square plates under a linearly decaying pressure pulse was also investigated [16].

The previous theoretical studies on the saturated impulse are all based on the time-independent deformation mechanism containing stationary plastic hinges; in other words, they are restricted to relevant modal solutions. Differently, the present paper will investigate the transient dynamic response of a fully clamped square plate subjected to a LRED pressure pulse, which is more complex and closer to the blast loading. In the literature, the blast loading is approximately modelled as linearly decaying pulse, exponentially decaying pulse, triangular pulse, or LRED pulse in the theoretical investigations. Among those simplifications, the LRED pulse is the most verisimilar one.

To facilitate engineering design, many efforts have been devoted to developing the equivalent method (EM) of replacing the complex-shaped pulse by a rectangular pulse. Symonds [17] concluded that the final deflection of a free beam subjected to a concentrated force pulse only depends on the total impulse and peak load of the pulse. Hodge [18] pointed out that this conclusion is valid only for loading intensities far beyond the yield load. Youngdahl [19, 20] presented an equivalent method (Youngdahl EM). Two correlation parameters, i.e. the rectangular pulse impulse I_e and effective load p_e, were proposed to eliminate the pulse shape effects, and provided an empirical estimation of the structural response duration. Zhu et al. [21] confirmed that the pulse approximation method is able to eliminate pulse shape effects on the dynamic plastic bending response of beams, and the deviations caused by Youngdahl EM are within 15%. By using the bound theorems, Li and Jones [22] established the theoretical foundation of Youngdahl EM for the rigid-plastic responses of common structural members. Li and Meng [23] further demonstrated that Youngdahl EM is effective to the SDOF model that reckons in bending moment and shear force. Ren et al. [24] examined the applicability of the Youngdahl EM for a tensor skin, indicating that this method is applicable for both stable and geometrically unstable structures. However, the applicability of the Youngdahl EM on the large deflection dynamic plastic response of the structure which is dominated by the membrane force has not been examined.

As a matter of fact, we will see that the effective pulse proposed by Youngdahl EM and the saturated impulse explored in our studies represent the same physical phenomenon for the plates under pulse loadings (although the judgments on the ending time of the plastic deformation are different); that is, the plates' plastic deformation may cease notably before the applied loading pulse ends. The present paper compares three EMs by using both the rigid-plastic and elastic-plastic material models.

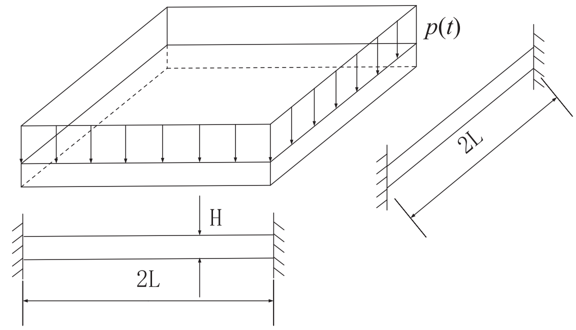

Fig. 1. A fully clamped square plate subjected to uniformly distributed pressure pulse.

2. R-PP analysis

2.1. Material properties and blast loading

As shown in Fig. 1, a fully clamped R-PP square plate with the side length of *2L* and thickness of *H* is considered. The plate is subjected to a LRED pressure pulse as sketched in Fig. 2 and the Linearly Rising Exponentially Decaying (LRED) pressure pulse has the following form:

$$p(t)=\begin{cases}\frac{p_0-3p_y}{t_0}t+3p_y & 0\le t\le t_0\\ p_0e^{-\frac{t-t_0}{\theta}} & t\ge t_0\end{cases} \tag{1}$$

where p_0 is the peak load of the LRED pressure pulse; p_y denotes the static collapse pressure of the fully clamped square plate ($p_y=12M_0/L^2$); t_0 is the duration of the pulse; and θ is a parameter with unit *s*. As shown in Fig. 2, $t_0+\theta$ is the intercept of the tangent of exponentially decaying phase through the point (t_0, p_0). According to a study by Cox and Morland [25] on the square plate subjected to uniformly distributed rectangular pressure pulse with the applied pressure p_0, the deformation mode of a plate under medium pulse load ($p_y<p_0\le 2p_y$) is identical to the static collapse mechanism, while the transient stage involves a