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Traveling wavefronts in a reaction–diffusion model with chemotaxis and nonlocal delay effect

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ABSTRACT

This paper is devoted to the study of traveling wavefronts of large wave speed for a reaction–diffusion model with chemotaxis and nonlocal delay effect by applying the perturbation method. The proof relies on an abstract formulation of the wave profile as a solution of an operator equation in a certain Banach space, coupled with the Fredholm theory and the Banach contraction mapping principle. This result is illustrated by an application to the chemotaxis–diffusion–growth model with the logistic source and a single delay effect.

1. Introduction

The prototypical chemotaxis model was proposed by Keller and Segel [1,2] to describe the aggregation of cellular slime molds *Dictyostelium discoideum* in response to the chemical cyclic adenosine monophosphate. Later, Keller and Segel [3] gave an explicit traveling wave solution by using the theoretical analysis. Since then a vast number of theoretical results have been developed for the Keller–Segel-type model [4–8]. For example, Nagai and Ikede [9] investigated the existence of traveling waves for a general Keller–Segel model. Note that the potential role of cell kinetics is ignored in the Keller–Segel model. This may be reasonable when cell proliferation has halted or the time-scale of movement is significantly faster than that of cell growth. When the diffusion rate and the chemotactic rate are both very small compared with the growth rate, however, the inclusion of cell proliferation and cell reduction becomes natural [10,11]. For this reason, Mimura and Tsujikawa [12] derived a population model including diffusion, chemotaxis and growth to describe the aggregating pattern dynamics of biological individuals. Usually, the functional representation of cell growth/death varies according to the biological system under consideration. Tello and Winkler [13] studied the Mimura–Tsujikawa model with the logistic source and obtained infinitely many local branches of nonconstant stationary solutions bifurcating from a positive constant solution. Winkler [14,15] studied the existence and boundedness of very weak global solutions. Ou and Yuan [16] investigated the existence

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of traveling wavefronts for the Mimura–Tsujikawa model when the growth function is a Fisher-type logistic source. Other study on traveling wave solutions of the chemotaxis model can be found in [17–19].

In [12–16], the growth function is assumed to be governed by a principle of causality, that is, the future state of the growth function is independent of the past states and is determined solely by the present. Under closer scrutiny, however, it becomes apparent that the principle of causality is often only a first approximation to the true situation and that a more realistic model would include some of past states of the growth function [20], for example, the growth of human tumor cells [21–23], hematopoietic stem cell [24–26], and *Dictyostelium discoideum* [27]. Moreover, in reality individuals sometimes compete for resource not only in their immediate neighborhood but also in a more board domain (see, for example, [28]). Hence, in this paper, we propose the following reaction–diffusion model with chemotaxis and nonlocal delay effect

$$\begin{cases} \dfrac{\partial u(x,t)}{\partial t} = d_1 \Delta u(x,t) - \alpha \nabla \cdot [u(x,t) \nabla \rho(x,t)] \\ \qquad\qquad + u(x,t) f\left(\displaystyle\int_{\mathbb{R}^N} \mathrm{d}\mu(y) u(x+y, t-\tau) \right), \\ \dfrac{\partial \rho(x,t)}{\partial t} = d_2 \Delta \rho(x,t) + a u(x,t) - b \rho(x,t), \end{cases} \tag{1.1}$$

where $x \in \mathbb{R}^N$ is the spatial variable; $t \geq 0$ is the time; $u(x,t)$ and $\rho(x,t)$ denote the population density of biological individuals and the concentration of a chemical substance at position x and time t, respectively; Δ and ∇ are the Laplacian and the gradient operators with respect to x; d_1 and d_2 are the diffusion rates of u and ρ, respectively; the term $-\alpha \nabla \cdot (u(x,t) \nabla \rho(x,t))$ denotes chemotaxis, and α is the chemotactic sensitivity; the nonlocal delay term $u(x,t) f(\int_{\mathbb{R}^N} \mathrm{d}\mu(y) u(x+y, t-\tau))$ denotes the proliferation and the reduction due to the death of the individuals and $\int_{\mathbb{R}^N} \mathrm{d}\mu(y) u(x+y, t-\tau)$ may account for all the competitive pressure when an individual born at location y can survive the immature period $[0, \tau]$ and has moved to location x when becoming mature (τ time units after birth); the terms $-b\rho(x,t)$ and $au(x,t)$ denote the degradation and the production of the chemical substance, respectively; d_1, d_2, a, b, τ and α are positive constants; f is C^k-smooth function for some $k \geq 2$; μ is a bounded measure on $\mathbb{R}^N$ and satisfies

$$\int_{\mathbb{R}^N} \mathrm{d}\mu(y) = 1 \qquad \text{and} \qquad \overline{\mu} \triangleq \int_{\mathbb{R}^N} \|y\|_{\mathbb{R}^N} \mathrm{d}|\mu(y)| < \infty.$$

The main goal of this paper is to study the existence of traveling wavefronts of system (1.1). Compared with the Mimura–Tsujikawa system, the only difference is that the growth function of system (1.1) has nonlocal delay effect. It should be mentioned that Ou and Yuan [16] investigated the existence of traveling wavefronts for system (1.1) without nonlocal delay effect, i.e., $\tau = 0$ and $\mathrm{d}\mu(y) = \delta(y)\mathrm{d}y$, where $\delta(\cdot)$ is the Dirac delta function. It is natural to ask whether the nonlocal delay effect has any influence on the existence of traveling wavefronts. In this paper, we shall give a positive answer to this question (see Theorem 1.1), that is, there exists a fixed positive constant τ^* such that system (1.1) has traveling wavefronts if and only if the delay $\tau \leq \tau^*$. This conclusion gives theoretical support to the actual biological experiment.

On the other hand, besides the literature given above, there have been some successful methods in establishing the existence of traveling wavefronts for the reaction–diffusion model (1.1) with zero chemotactic sensitivity ($\alpha = 0$), for example, the methods of the super/subsolution pair [29], homotopy [30], Leray–Schauder degree [31], monotone iterations [32] and perturbation [33]. To the best of our knowledge, however, no rigorous work has previously been done for the existence of traveling wavefronts in reaction–diffusion equations like system (1.1) with both chemotaxis and nonlocal delay effect.

In this paper, we study the existence of traveling wavefront solutions for system (1.1) by the perturbation method. This approach is based on an abstract formulation of the wave profile as a solution of an operator equation in a certain Banach space, coupled with an index formula of the associated Fredholm operator, some careful estimation of the nonlinear perturbation and the Banach contraction mapping principle. This approach relates the existence of traveling wavefront solutions to the existence of heteroclinic connecting orbits of an associated functional differential equation of system (1.1). This idea has been exploited in the literature (see e.g. [33]). When compared with [33], the major novelty in the present paper is that the existence conditions of heteroclinic solutions of the reaction equation (2.1) will be presented explicitly.