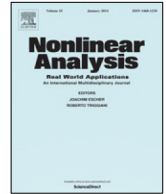




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# Allee effect acting on the prey species in a Leslie–Gower predation model



Nicole Martínez-Jeraldo, Pablo Aguirre\*

Departamento de Matemática, Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile

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## ABSTRACT

We study the consequences of an Allee effect acting on the prey species in a Leslie–Gower predator–prey model. For this goal we make extensive use of analytical tools from dynamical systems theory complemented with a numerical bifurcation analysis. By studying the dynamics at infinity under a suitable compactification we prove that the model is well-posed in the sense that all the solutions are bounded. We provide a thorough analysis of the number and stability of equilibrium points. In particular, the origin is a non-hyperbolic equilibrium and presents different regimes of local (un)stability depending on certain conditions on the model parameters. In addition, we find curves of homoclinic, Hopf, and saddle–node bifurcations around a Bogdanov–Takens point. In this process, our findings indicate that the survival threshold for both populations in the two-dimensional phase space can be either a limit cycle, a homoclinic orbit, or the stable manifold of a saddle–equilibrium.

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The dynamic relationship between two populations – a prey species and a predatory species – has been an important topic in the mathematical ecology field for a long time; see for instance [1–3] and the references therein. In particular, special attention has been paid to the phenomenon of Allee effect on one or more populations [4–7]. The Allee effect acting on a given population is commonly understood as the tendency of such species to face difficulties to grow from low densities, and to avoid extinction. This phenomenon has been observed as the per-capita population growth rate tends to decrease under some minimum critical level [8]. Indeed, the per-capita population growth rate may become negative indicating the existence of an extinction threshold – commonly known as the Allee threshold – that the population has to overcome in order to survive and avoid extinction [8–10].

In this work we model a predator–prey system of Leslie–Gower type [11,12] in which the prey population is affected by an Allee effect [13–15]. The model is described by the following two-dimensional system of

\* Corresponding author.

E-mail addresses: [nicole.martinez.14@sansano.usm.cl](mailto:nicole.martinez.14@sansano.usm.cl) (N. Martínez-Jeraldo), [pablo.aguirre@usm.cl](mailto:pablo.aguirre@usm.cl) (P. Aguirre).

nonlinear differential equations:

$$X_\mu : \begin{cases} \frac{dx}{dt} = rx \left(1 - \frac{x}{K}\right) \left(1 - \frac{m+b}{x+b}\right) - qxy; \\ \frac{dy}{dt} = sy \left(1 - \frac{y}{nx}\right); \end{cases} \quad (1)$$

where  $x = x(t)$  and  $y = y(t)$  are the population sizes of prey and predator, respectively, for  $t \geq 0$ , and where  $\mu = (r, K, q, s, n, b, m) \in E := \mathbb{R}_+^6 \times ]-K, K[$  is a vector of parameters. In particular, parameters  $r$  and  $s$  represent the intrinsic growth rate of prey and the predator, respectively,  $K$  is the environmental carrying capacity of the prey, and  $q$  is the maximum per capita consumption rate of predators. We explain the interpretation of the different terms and remainder parameters in (1) as follows. The model (1) is of Leslie–Gower type; its fundamental characteristic is that the equation describing the growth of predators is of logistic type [12], where the environmental carrying capacity of the predators  $K_y = nx$  is proportional to the number of prey available. In other words, the model assumes that predators are specialists. This Leslie–Gower modelling approach contrasts with Gause-type models in which conversion of prey into predator densities due to predation follows a sort of conservation law [16–18]. The model (1) also considers that the functional response is linear [12], but different ones can be incorporated, according to the classification proposed by C. S. Holling [11], as has been the case in other models [19–22].

As we mentioned before, the prey population is affected by an Allee effect. The Allee effect occurs when the fitness of an individual, in a small or sparse population, decreases as population size or density also decreases [10,15]. In general, two types of Allee effect are usually distinguished: the strong Allee effect implies a critical population size (also called the “Allee threshold”) under which the population growth rate becomes negative [13,23]. On the other hand, the weak Allee effect implies a reduced (non-negative) per capita growth rate at low population densities [13,23]. There are different mechanisms that can generate Allee effect, for instance, the difficulty of finding a mate among individuals of the same species at low population densities (see Table 1 in [24]). Different ecological investigations have suggested that two or more Allee effects can produce mechanisms that act simultaneously on a single population. Such combination is called a multiple Allee effect [24]. In (1) the Allee effect is given by the term  $A(x) = 1 - \frac{m+b}{x+b}$  which multiplies the expression of classic logistic growth in the prey equation. This form for  $A(x)$  was proposed in [25] and accounts for a higher extinction rate on small populations than in other modelling approaches. Here,  $b > 0$  and  $-b < m < K$ , so that  $m + b > 0$ . In the absence of predators, if  $0 < m < K$  system (1) shows strong Allee effect; if  $\frac{-bK}{b+K} < m < 0$ , we have weak Allee effect; and if  $-b < m < \frac{-bK}{b+K}$ , (1) has a logistic growth.

The consequences of the Allee effect for ecology and conservation have been vastly investigated; see [6,10] and the references therein as entry points on the subject. For instance, the same expression for  $A(x)$  as in (1) has been considered to compare its properties to predation models with other mathematical formulations of Allee effect [26,27]. The aim of this work is to extend the analysis to include a Leslie–Gower formulation and to address the question of what are the possible survival thresholds in the model, i.e., those boundaries in phase space that determine whether a given initial condition leads to extinction or longterm subsistence. Our study of (1) is carried out with tools from the theory of dynamical systems. By means of topological equivalences, time rescalings and appropriate parameter changes, we form an extended polynomial vector field, which is topologically equivalent to the initial model, on which we perform the analysis. By studying the dynamics at infinity we prove that the solutions of the model are bounded. We also find that the origin is a non-hyperbolic equilibrium and describe different scenarios of (un)stability that depend (mainly) on parameters  $r$ ,  $b$  and  $m$ . In addition, we determine the number and stability of all the equilibrium points, and –using both analytical and numerical techniques – we find the bifurcation diagram including local and global bifurcations. In particular we find curves of homoclinic, Hopf, and saddle–node bifurcations around a Bogdanov–Takens point. In this process, we identify the survival threshold in the two-dimensional phase

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