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Chuan-Fu Yang, Dai-Quan Liu

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HALF-INVERSE PROBLEM FOR THE DIRAC OPERATOR

CHUAN-FU YANG, DAI-QUAN LIU,

Abstract. Given the spectrum of the Dirac operator, together with the potential on the half-interval and one boundary condition, this paper provides reconstruction of the potential on the whole interval, and proves the existence conditions of the solution.

1. Introduction

Let us consider the canonical form of the Dirac operator

$$By' + Q(x)y = \lambda y, \quad 0 < x < 1 \tag{1.1}$$

with

$$B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},\ Q(x) = \begin{pmatrix} p(x) & q(x) \\ q(x) & -p(x) \end{pmatrix},\ y(x) = \begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix},$$

subject to the boundary conditions

$$y_1(0)\cos\alpha + y_2(0)\sin\alpha = 0, \tag{1.2}$$

$$y_1(1)\cos\beta + y_2(1)\sin\beta = 0. \tag{1.3}$$

Here $p(\cdot), q(\cdot) \in L^2(0,1)$, and $0 \le \alpha, \beta < \pi$.

Denote by $\varphi(x,\lambda) = (\varphi_1(x,\lambda), \varphi_2(x,\lambda))^T$ the solution of Eq.(1.1) with the initial-value condition $\varphi(0,\lambda) = (\sin\alpha, -\cos\alpha)^T$, then there exists (see [2, 8]) kernel matrix $K(x,t) = (K_{ij}(x,t))_{i,j=1}^2$ with entries continuously differentiable on $0 \le t \le x \le 1$ such that

$$\varphi(x,\lambda) = \varphi_0(x,\lambda) + \int_0^x K(x,t)\varphi_0(t,\lambda)dt, \tag{1.4}$$

where $\varphi_0(x,\lambda) = (\sin(\lambda x + \alpha), -\cos(\lambda x + \alpha))^T$.

The characteristic function is defined by

$$\Phi(\lambda) = \varphi_1(1,\lambda)\cos\beta + \varphi_2(1,\lambda)\sin\beta. \tag{1.5}$$

Taking Eq.(1.4) into account, one has

$$\Phi(\lambda) = \sin(\lambda + \alpha - \beta) + \psi(\lambda),$$

where $\psi \in \mathcal{L}^1$ ($\mathcal{L}^a$—the class of the entire functions of exponential type $\le a$ which belong to $L_2(\mathbb{R})$ for real λ).

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