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## HALF-INVERSE PROBLEM FOR THE DIRAC OPERATOR

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ABSTRACT. Given the spectrum of the Dirac operator, together with the potential on the half-interval and one boundary condition, this paper provides reconstruction of the potential on the whole interval, and proves the existence conditions of the solution.

### 1. INTRODUCTION

Let us consider the canonical form of the Dirac operator

$$By' + Q(x)y = \lambda y, \quad 0 < x < 1 \quad (1.1)$$

with

$$B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad Q(x) = \begin{pmatrix} p(x) & q(x) \\ q(x) & -p(x) \end{pmatrix}, \quad y(x) = \begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix},$$

subject to the boundary conditions

$$y_1(0) \cos \alpha + y_2(0) \sin \alpha = 0, \quad (1.2)$$

$$y_1(1) \cos \beta + y_2(1) \sin \beta = 0. \quad (1.3)$$

Here  $p(\cdot), q(\cdot) \in L^2(0, 1)$ , and  $0 \leq \alpha, \beta < \pi$ .

Denote by  $\varphi(x, \lambda) = (\varphi_1(x, \lambda), \varphi_2(x, \lambda))^T$  the solution of Eq.(1.1) with the initial-value condition  $\varphi(0, \lambda) = (\sin \alpha, -\cos \alpha)^T$ , then there exists (see [2, 8]) kernel matrix  $K(x, t) = (K_{ij}(x, t))_{i,j=1}^2$  with entries continuously differentiable on  $0 \leq t \leq x \leq 1$  such that

$$\varphi(x, \lambda) = \varphi_0(x, \lambda) + \int_0^x K(x, t)\varphi_0(t, \lambda)dt, \quad (1.4)$$

where  $\varphi_0(x, \lambda) = (\sin(\lambda x + \alpha), -\cos(\lambda x + \alpha))^T$ .

The characteristic function is defined by

$$\Phi(\lambda) = \varphi_1(1, \lambda) \cos \beta + \varphi_2(1, \lambda) \sin \beta. \quad (1.5)$$

Taking Eq.(1.4) into account, one has

$$\Phi(\lambda) = \sin(\lambda + \alpha - \beta) + \psi(\lambda),$$

where  $\psi \in \mathcal{L}^1$  ( $\mathcal{L}^a$ —the class of the entire functions of exponential type  $\leq a$  which belong to  $L_2(\mathbb{R})$  for real  $\lambda$ ).

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