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Stability of highly nonlinear hybrid stochastic integro-differential delay equations

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ABSTRACT

For the past few decades, the stability criteria for the stochastic differential delay equations (SDDEs) have been studied intensively. Most of these criteria can only be applied to delay equations where their coefficients are either linear or nonlinear but bounded by linear functions. Recently, the stability criterion for highly nonlinear hybrid stochastic differential equations is investigated in Fei et al. (2017). In this paper, we investigate a class of highly nonlinear hybrid stochastic integro-differential delay equations (SIDDEs). First, we establish the stability and boundedness of hybrid stochastic integro-differential delay equations. Then the delay-dependent criteria of the stability and boundedness of solutions to SIDDEs are studied. Finally, an illustrative example is provided.

1. Introduction

In many real-world systems, such as science, industry, economics and finance etc., we will encounter a time delay. The differential delay equations (DDEs) including the functional differential equations have been used to model such time-delay systems. Since the time-delay often causes the instability of systems, stability of DDEs has been researched intensively for more than 50 years. Generally, the stability criteria are classified into the delay-dependent and delay-independent stability criteria. When the size of delays is incorporated into the delay-dependent stability criteria, the delay-dependent systems are generally less conservative than the delay-independent ones which work for any size of delays. There exists a very rich literature in this area (see, e.g., [1–5]).

In 1980’s, the stochastic differential delay equations were developed in order to model the real-world systems which are subject to external noises (see, e.g., [6]). Since then, in the study of SDDEs the stability has been one of the most important topics (see, e.g., [7–12]).

Since 1990’s, the hybrid SDDEs (called also SDDEs with Markovian switching) were developed to model the real-world systems where they may experience abrupt changes in their parameters and structures in addition to uncertainties and time lags. One of the important issues in the study of hybrid SDDEs is the analysis of stability of control systems. Moreover, the delay-dependent stability criteria have been created by many authors (see, e.g., [13–20]). To our best knowledge, the existing delay-dependent stability criteria are mainly provided for the hybrid SDDEs where their coefficients are either linear or nonlinear but bounded by linear functions (or, satisfy the linear growth condition). Recently, [21,22] initiate the investigation on the stability of the hybrid highly nonlinear stochastic delay differential equations. Based on the highly nonlinear hybrid SDDEs (see, e.g., [21,22]), the stability of highly nonlinear systems is further explored in [23–27]. However, the current states of many real systems depend on several history states of some time interval. Thus multiple time delay systems are also discussed (see, e.g., [28,29]).

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On the other hand, a real system depends on not only discrete delays (single or multiple ones) but also a whole history of states of the system with some lag interval. [30] considers a nonhomogeneous Volterra integro-differential equation with the solution being a non-Markovian process. Moreover, the convergence and stability of the linear stochastic integro-differential delay equations were discussed in [31–40]. With the energy of a hybrid system accumulated, a stable system might get unstable as it is disturbed by a white noise. In general, if a stable system has too long lag, then it might get unstable. Our problem is as follows: In how long lag time, the system can remain stable? To this end, we discuss an example as follows. Now we consider the stability of the following hybrid highly nonlinear SIDDE

$$dX(t) = \begin{cases} (-10X^3(t) - 2\hbar(X_t))dt + (\hbar(X_t))^2 dB(t), & \text{if } i = 1, \\ (\hbar(X_t) - 5X^3(t))dt + (\hbar(X_t))^2 dB(t), & \text{if } i = 2. \end{cases} \tag{1.1}$$

Here for $\tau \geq 0$ $X_t := \{X(t+u) : -\tau \leq u \leq 0\}$, $\hbar(X_t) := \frac{1}{\tau}\int_{-\tau}^{0} X(t+u)du$ with $\hbar(X_t) := X(t)$ for $\tau = 0$, $X(t) \in \mathbb{R}$ is the state of the highly non-linear hybrid system, $B(t)$ is a scalar Brownian motion, $r(t)$ is a Markovian chain with the state space $\mathbb{S} = \{1, 2\}$ and its generator Γ given by

$$\Gamma = \begin{pmatrix} -1 & 1 \\ 8 & -8 \end{pmatrix}. \tag{1.2}$$

The above system (1.1) will switch from one mode to the other according to the probability law of the Markovian chain. If the time delay $\tau = 0.01$, the computer simulation shows it is asymptotically stable (see Fig. 4.1). If the time-delay is large, say $\tau = 3$, the computer simulation shows that the hybrid SIDDE (1.1) is unstable (see Fig. 4.2). In other words, whether the hybrid SIDDE is stable or not depends on how small or large the time-delay is. On the other hand, both drift and diffusion coefficients of the hybrid SIDDE affect the stability of the systems due to high nonlinearity. However, there is no delay dependent criterion which can be applied to the SIDDE to derive a sufficient bound on the time-delay τ such that the SIDDE is stable, although the stability criteria of the highly nonlinear hybrid SDDE have been discussed in [23] on the single delay. The aim of this paper is to establish the delay dependent criteria for the highly nonlinear hybrid SIDDEs.

Our main contributions are as follows:

(a) The hybrid highly nonlinear stochastic integro-differential delay equations first are investigated, where the coefficients are highly nonlinear on both the current state $X(t)$ and the history $\hbar(X_t)$ with lag time $\tau \geq 0$.

(b) We established the theorem of the stability and boundedness of the solutions to the hybrid highly nonlinear SIDDEs similar to [21] where they only investigate the highly nonlinear hybrid SDDE (see Theorem 2.4 in Section 2 below).

(c) The delay-dependent criteria are established first for the solutions to the hybrid highly nonlinear SIDDEs in Section 3.

(d) New mathematical techniques are well applied to solve our stability criteria, such as by constructing an appropriate Lyapunov functional.

2. Notation and assumptions

Throughout this paper, unless otherwise specified, we use the following notation. If A is a vector or matrix, its transpose is denoted by $A^\top$. If $x \in \mathbb{R}^d$, then $|x|$ is its Euclidean norm. For a matrix A, we let $|A| = \sqrt{\text{trace}(A^\top A)}$ be its trace norm and $\|A\| = \max\{|Ax|: |x| = 1\}$ be the operator norm. Let $\mathbb{R}_+ = [0, \infty)$. Denote by $C([-\tau, 0]; \mathbb{R}^d)$ the family of continuous functions η from $[-\tau, 0] \to \mathbb{R}^d$ with the norm $\|\eta\| = \sup_{-\tau \leq u \leq 0}|\eta(u)|$. If A is a subset of Ω, denote by I_A its indicator function. Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions. Let $B(t) = (B_1(t), \ldots, B_m(t))^\top$ be an m-dimensional Brownian motion defined on the probability space. Let $r(t)$, $t \geq 0$, be a right-continuous Markov chain on the probability space taking values in a finite state space $\mathbb{S} = \{1, 2, \ldots, N\}$ with generator $\Gamma = (\gamma_{ij})_{N\times N}$ given by

$$\mathbb{P}\{r(t+\Delta) = j | r(t) = i\} = \begin{cases} \gamma_{ij}\Delta + o(\Delta) & \text{if } i \neq j, \\ 1 + \gamma_{ii}\Delta + o(\Delta) & \text{if } i = j, \end{cases}$$

where $\Delta > 0$. Here $\gamma_{ij} \geq 0$ is the transition rate from i to j if $i \neq j$ while $\gamma_{ii} = -\sum_{j\neq i}\gamma_{ij}$. We assume that the Markov chain $r(\cdot)$ is independent of the Brownian motion $B(\cdot)$. Let

$$f(\cdot,\cdot,\cdot) : \mathbb{R}^d \times \mathbb{S} \times \mathbb{R}_+ \to \mathbb{R}^d, \qquad g(\cdot,\cdot,\cdot) : \mathbb{R}^d \times \mathbb{S} \times \mathbb{R}_+ \to \mathbb{R}^{d\times m}$$

$$F(\hbar(\cdot),\cdot,\cdot) : C([-\tau, 0]; \mathbb{R}^d) \times \mathbb{S} \times \mathbb{R}_+ \to \mathbb{R}^d, \qquad G(\hbar(\cdot),\cdot,\cdot) : C([-\tau, 0]; \mathbb{R}^d) \times \mathbb{S} \times \mathbb{R}_+ \to \mathbb{R}^{d\times m}$$

be Borel measurable functions, where $\hbar(\phi) := \frac{1}{\tau}\int_{-\tau}^{0}\phi(u)du$, $\phi \in C([-\tau, 0]; \mathbb{R}^d)$ with $\hbar(\phi) := \phi(0)$ for $\tau = 0$. Let the functional $X_t := \{X(t+u) : -\tau \leq u \leq 0\}$. Consider a d-dimensional hybrid highly nonlinear SIDDE with Assumption 2.1 below

$$\begin{aligned} dX(t) = &[f(X(t), r(t), t) + F(\hbar(X_t), r(t), t)]dt \\ &+ [g(X(t), r(t), t) + G(\hbar(X_t), r(t), t)]dB(t) \end{aligned} \tag{2.1}$$

on $t \geq 0$ with initial data

$$\{\eta(t) : -\tau \leq t \leq 0\} = \eta \in C([-\tau, 0]; \mathbb{R}^d), \ r(0) = i_0 \in \mathbb{S}. \tag{2.2}$$