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Stability of symmetry breaking states in finite-size Dicke model with photon leakage

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ABSTRACT

We investigate the finite-size Dicke model with photon leakage. It is shown that the symmetry breaking states, which are characterized by non-vanishing $\langle \hat{a} \rangle \neq 0$ and correspond to the ground states in the superradiant phase in the thermodynamic limit, are stable, while the eigenstates of the isolated finite-size Dicke Hamiltonian conserve parity symmetry. We introduce and analyze an effective master equation that describes the dynamics of a pair of the symmetry breaking states that are the degenerate lowest energy eigenstates in the superradiant region with photon leakage. It becomes clear that photon leakage is essential to stabilize the symmetry breaking states and to realize the superradiant phase without the thermodynamic limit. Our theoretical analysis provides an alternative interpretation using the finite-size model to explain results from cold atomic experiments showing superradiance with the symmetry breaking in an optical cavity.

1. Introduction

The Dicke model is one of the quantum optical models that has been thoroughly studied [1,2]. It describes a collection of identical two-level atoms that are coupled with a single electromagnetic mode in a cavity via a dipole interaction. The significant property of the Dicke model is that it exhibits a transition from a normal phase to a superradiant phase when the coupling constant takes a critical value in the thermodynamic limit [3–6]. Since it is known that this phase transition occurs even at zero temperature, it is considered to be a quantum phase transition [5,6]. Thanks to recent experimental progress in atomic physics, the situations described by the Dicke model have been realized in cold atomic systems in an optical cavity [7,8], where the collection of cold atoms plays the same role as a collection of two-level atoms. In these experiments the transition to a superradiant phase is verified by detecting photons leaking from the cavity. However, this transition cannot be identified to be a quantum transition and/or thermal transition that is defined in equilibrium infinite-size systems, since in the cold atom experiments the number of atoms is finite and the system is open. Instead, it is suggested that the transition in the experiments can be interpreted as a nonequilibrium phase transition [7,9,10], the photon leaking being taken into account in the thermodynamic limit. To our best knowledge, this dissipative Dicke model has been investigated in a semi-classical and (plus) stochastic approach based on the thermodynamic limit. The semi-classical approach [7,11] implies that quantum operators are replaced with c-numbers and that the superradiant transition is described as a bifurcation of the classical solution. We point out that this treatment ignores quantum fluctuation, *i.e.*, the quantum entanglement between the atoms and the cavity mode. As the stochastic method [11], a stochastic term that represents a dissipation is added to the Heisenberg equation in such a phenomenological manner that the stochastic operator of the bosonic quasi-particle defined in each of normal and superradiant phases is introduced. Strictly speaking, this quasi-particle picture is exact only in the thermodynamic limit, where the Hamiltonian becomes a corresponding quadratic form. Thus this stochastic method is valid only when the system is close to the thermodynamic limit. When a finite-size system is under consideration instead of the thermodynamic limit, the higher order terms in the Hamiltonian that were neglected in the thermodynamic limit may affect the quasi-particle picture and the gap in theoretical treatment depending on whether the phase is either normal or superradiant is unfavorable. Hence, it is still not clear how the superradiant transition is explained in the finite-size model with the dissipation. Our analysis in this paper focuses on a stability of the superradiant state, without relying on the thermodynamic limit and the quasi-particle picture established then and taking account of quantum fluctuations properly.

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For the isolated finite-size Dicke model, there are some previous studies on the singularity in the ground state energy associated with the superradiant phase transition or its finite-size corrections [12–14]. There the model is treated as an isolated system without symmetry breaking. In distinction from the previous research, we introduce the interaction of the finite-size Dicke system with an environment in this paper and focus on the mechanism to realize the symmetry breaking state, which is characterized by $\langle\hat{a}\rangle \neq 0$, as observed in the experiments [7,8].

The purpose of this paper is to show that the Dicke model with photon leakage exhibits symmetry breaking, even in finite-size systems. To achieve this, restricting ourselves to small atomic level spacing, we first study the ground and first excited states of the isolated Dicke model and estimate an energy gap between them, because the two states form a pair of degenerate states when the superradiant phase is realized. Then, we introduce photon leakage out of the cavity to an external vacuum and investigate the temporal evolution of the density matrix in the superradiant region. It will be shown that the symmetry breaking state becomes stable. Thus, photon leakage is crucial for understanding symmetry breaking in a finite-size Dicke system.

This paper is organized as follows. In Sec. 2, we briefly introduce the Dicke model and compare the finite-size model with the model in the thermodynamic limit. The lowest energy eigenstates are constructed under a "polarization condition", and we have a pair of the two almost degenerate states, breaking the parity symmetry, in the superradiant region in Sec. 3. Section 4 shows that, although the symmetry breaking states are not exact eigenstates, they freeze dynamically. Considering the leakage photon, we derive and analyze an effective master equation for the two symmetry breaking states in the open Dicke model that interacts with the environment in Sec. 5 and discuss the stability of the symmetry breaking states.

2. Parity symmetry and superradiance

The Dicke Hamiltonian is given by [3–6]

$$\hat{H}_{\mathrm{DH}} = \omega_0 \hat{J}^{(3)} + \omega \hat{a}^\dagger \hat{a} + \lambda \varphi_{\mathrm{c}} \left[\left(\hat{a} + \hat{a}^\dagger \right) \left(\hat{J}^+ + \hat{J}^- \right) \right], \tag{1}$$

where $\hat{a}$ denotes the bosonic annihilation operator for the cavity mode with frequency ω, and $\hat{J}^{(i)}$ $(i = 1, 2, 3)$ are pseudospin operators describing a collection of N identical two-level atoms with level spacing ω_0; these operators obey angular momentum algebra. The $\hat{J}^{\pm}$ operators are defined by $\hat{J}^{(1)} \pm i \hat{J}^{(2)}$, and, we take $J = N/2$ for the length of the pseudospin J. The coefficients ω_0, ω, and λ are non-negative. The symbol φ_{c} stands for the normalization factor of wave function for cavity mode [3], namely $\varphi_{\mathrm{c}} = 1/\sqrt{2J}$. Note that we do not neglect the counter-rotating contributions in the Hamiltonian (1). As in Ref. [13], we employ the following Hamiltonian, transformed by the unitary operator $\hat{U} = \exp\left[i(\pi/2) \hat{J}^{(2)} \right]$,

$$\hat{H} = \hat{U} \hat{H}_{\mathrm{DH}} \hat{U}^\dagger = -\omega_0 \hat{J}^{(1)} + \omega \hat{a}^\dagger \hat{a} + 2\lambda \varphi_{\mathrm{c}} \left(\hat{a} + \hat{a}^\dagger \right) \hat{J}^{(3)}, \tag{2}$$

because the diagonalized form of the interaction term is convenient for our arguments. The parity transformation in this representation is executed by the unitary operator

$$\hat{\Pi} = \exp\left[i \left(\hat{a}^\dagger \hat{a} - \hat{J}^{(1)} \right) \pi \right], \tag{3}$$

and $\hat{H}$ is invariant under the parity transformation $\hat{\Pi}$, namely $[\hat{H}, \hat{\Pi}] = 0$.

In the thermodynamic limit, where $N \to \infty$, the system shows two phases that separated by the critical coupling constant $\lambda_c = \sqrt{\omega_0 \omega}/2$ [5,6]. For $\lambda < \lambda_c$, the system is in the normal phase, where the eigenstates are symmetric, that is, they each have a definite parity. For $\lambda > \lambda_c$, the system is in the superradiant phase where the atoms are collectively excited and the light field obtains a coherent amplitude. The ground state in the superradiant phase breaks the parity symmetry [6], which means that the generation of the superradiant phase is interpreted as a spontaneous symmetry breaking with the nonvanishing order parameter $\langle\hat{a}\rangle \neq 0$.

In the finite-size model that we will focus on, the ground and first excited state form a degenerate pair in the superradiant region [14,15], which is characterized by closing the energy gap between them. It is also reported that the similar pair formation presents in the higher excited states [15,16]. These degeneracies occur asymptotically as λ increases. It is, as will be shown, essential to form symmetry breaking states in the superradiant region.

3. Formation of a degenerate pair in superradiant region

To investigate a degenerate pair with the two lowest states analytically, we will construct the ground and first excited states in a perturbative manner. For convenience, we introduce the scaled Hamiltonian $\bar{H}$,

$$\bar{H} = -\bar{\omega}_0 \hat{J}^{(1)} + \bar{\omega} \hat{a}^\dagger \hat{a} + \left(\hat{a} + \hat{a}^\dagger \right) \hat{J}^{(3)}, \tag{4}$$

where

$$\bar{\omega}_0 = \frac{\omega_0}{2\lambda \varphi_{\mathrm{c}}}, \quad \bar{\omega} = \frac{\omega}{2\lambda \varphi_{\mathrm{c}}}. \tag{5}$$

First, we consider the limiting case of $\bar{\omega}_0 = 0$, keeping $\bar{\omega}$ finite. In this limit, we can construct all the eigenstates in the following way. Since $[\bar{H}, \hat{J}^{(3)}] = 0$, we can represent $\bar{H}$ in each subspace, labeled by the eigenvalue m of $\hat{J}^{(3)}$, as

$$\bar{H}_m = \bar{\omega} \hat{a}_m^\dagger \hat{a}_m - \frac{m^2}{\bar{\omega}}, \tag{6}$$

where $\hat{a}_m = \hat{a} + d_m$, $d_m = m/\bar{\omega}$. Then, the eigenstates of $\bar{H}_m$ are exhausted by $|m\rangle \otimes \left(\hat{a}_m^\dagger \right)^n |0_m\rangle / \sqrt{n!}$, where the coherent state $|0_m\rangle$ is defined by

$$\hat{a} |0_m\rangle = -d_m |0_m\rangle .$$

Obviously, there are two orthogonal ground states, $\left| \Psi_0^{(\pm J)} \right\rangle$,

$$\left| \Psi_0^{(\pm J)} \right\rangle = |\pm J\rangle \otimes \left| 0_{\pm J} \right\rangle. \tag{7}$$

This ensures ground state degeneracy in our limiting case.

Next, we revive the $\bar{\omega}_0$ term while restricting ourselves to the assumption $0 < \bar{\omega}_0 \ll 1$. Then, the $\bar{\omega}_0$ term can be regarded as a perturbation, which lifts the degeneracy of the two ground states at $\bar{\omega}_0 = 0$ at the $2J$ th-order. Since the non-degenerate eigenstate has well-defined parity, we have the ground state $+1$ (even) parity, $|\Psi_0\rangle$ and the first excited state -1 (odd) parity, $|\Psi_1\rangle$. Hence, they are given by

$$|\Psi_0\rangle = \frac{1}{\sqrt{2}} \left[\left| \Psi_0^{(-J)} \right\rangle + \left| \Psi_0^{(+J)} \right\rangle \right] + \mathcal{O}(\delta), \tag{8}$$

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}} \left[\left| \Psi_0^{(-J)} \right\rangle - \left| \Psi_0^{(+J)} \right\rangle \right] + \mathcal{O}(\delta), \tag{9}$$

since $\hat{\Pi} \left| \Psi_0^{(\pm J)} \right\rangle = \left| \Psi_0^{(\mp J)} \right\rangle$. Here, δ is estimated as $\delta = \bar{\omega}_0 \bar{\omega} / \sqrt{N} = \sqrt{N} \lambda_c^2 / \lambda^2$ in the leading order. For $\delta \ll 1$, or $\lambda \gg \lambda_c N^{1/4}$, which we will call polarization condition, the terms $\mathcal{O}(\delta)$ in the expressions (8) and (9) are negligible.