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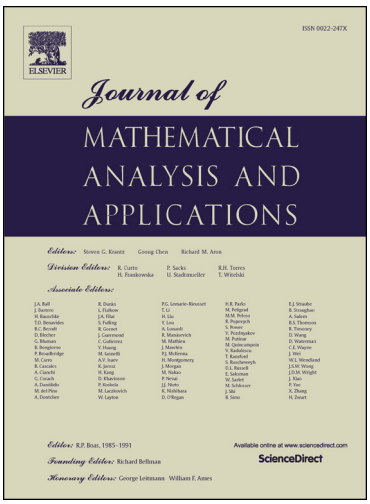

Asymptotic behavior and blow-up of solutions for infinitely degenerate semilinear parabolic equations with logarithmic nonlinearity

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Asymptotic behavior and blow-up of solutions for infinitely degenerate semilinear parabolic equations with logarithmic nonlinearity ☆

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Abstract

In this paper, we study the initial-boundary value problem for infinitely degenerate semilinear parabolic equations with logarithmic nonlinearity $u_t - \triangle_X u = u \log |u|$, where $X = (X_1, X_2, \cdots, X_m)$ is an infinitely degenerate system of vector fields, and $\triangle_X := \sum_{j=1}^m X_j^2$ is an infinitely degenerate elliptic operator. Using potential well method, we first prove the invariance of some sets and vacuum isolating of solutions. Then, by the Galerkin method and the logarithmic Sobolev inequality, we obtain the global existence and blow-up at $+\infty$ of solutions with low initial energy or critical initial energy, and we also discuss the asymptotic behavior of the solutions.

Keywords: Infinitely degenerate parabolic equation, global existence, blow-up, logarithmic nonlinearity

1. Introduction

For our purpose we first recall some notions, see [12, 25] for details. Let Ω' be an open domain in $\mathbb{R}^n$ $(n \geq 2)$, and $X = (X_1, \cdots, X_m)$ be a system of real smooth vector fields defined in Ω'. We assume that each X_i is a formally skew-adjoint operator, i.e. $X_i = -X_i^*$, where X_i^* is the formal adjoint operator of X_i in the sense of $(X_i^*\varphi, \psi)_{L^2(\Omega')} = (\varphi, X_i\psi)_{L^2(\Omega')}$, for any $\varphi, \psi \in C_0^\infty(\Omega')$. Denote $I = (i_1, i_2, \cdots, i_k)$, and $X_I = [X_{i_1}, [X_{i_2}, \cdots [X_{i_{k-1}}, X_{i_k}] \cdots]]$ as the k-th repeated commutators of $(X_1, X_2, \ldots, X_m)$, where $1 \leq i_j \leq m$, and $|I| = k$ is called the length of the commutators. We say that X satisfies Hörmander's condition in Ω' with Hörmander's index $Q < +\infty$, if there is an integer $Q \geq 1$ such that the vector fields $\{X_1, X_2, \ldots, X_m\}$ together with their commutators of length at most Q can span the tangent space at each point $x \in \Omega'$. If X satisfies the Hörmander's condition, we say that X is finite degenerate and the self-adjoint operator $\triangle_X := \sum_{j=1}^m X_j^2$ is called finitely degenerate elliptic operator, otherwise X is infinitely degenerate and the operator $\triangle_X$ is called an infinitely degenerate elliptic operator. Let $\Gamma = \bigcup_{j \in J} \Gamma_j$ be the union of a family of smooth surfaces in Ω', we say that Γ is non-characteristic for the system of vector fields X, if for any point $x_0 \in \Gamma$, there exists at least one vector field of $X_1, X_2, \cdots, X_m$ which traverses Γ_j at x_0 for all $j \in J_0 = \{k \in J;\ x_0 \in \Gamma_k\}$.

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