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Positive solutions of a nonlocal singular elliptic equation by means of a non-standard bifurcation theory

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ABSTRACT

We consider a nonlocal elliptic equation arising in a prey–predator model whose nonlocal term is singular. We use the Leray–Schauder degree to prove the existence of an unbounded continuum of positive solutions emanating from the trivial solution. As application, we study nonlocal and singular elliptic equations of the type logistic and Holling–Tanner.

1. Introduction

From the pioneering paper of Furter and Grinfeld [8], nonlocal terms have been included in population dynamics models in order to take into account that the variation of the species at a point depends not only on the behavior of the species at the point but in the entire environment, see also the recent reference [9] where a detailed study of models that come from the Biology is carried out. Specifically, in [7] and [12] a reaction–diffusion–chemotaxis predator–prey mathematical system is proposed to model the interacting of two populations, one of amoebae and one of virulent bacteria:

$$\begin{cases} u_t = D_1 \Delta u + u(1-u-v), \\ v_t = D_2 \Delta v - \chi \nabla \cdot (v \nabla u) - \mu v + \delta v \dfrac{\int_\Omega u(x)v(x)\,\mathrm{d}x}{\int_\Omega v(x)\,\mathrm{d}x} - \dfrac{\gamma u v}{1+\tau v}, \end{cases} \tag{1}$$

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in a habitat Ω, where Ω is a regular subset of $\mathbb{R}^N$, with $N = 1$ or $N = 2$. In (1), $u(x,t)$ and $v(x,t)$ denote the concentration of bacteria and amoebae at time t and position x, respectively. The numbers D_1 and D_2 represent the diffusion rate of bacteria and amoebae, respectively. χ is a chemotactic coefficient and μ is the natural mortality rate of amoebae. The last term of the second equation of (1) is due to the fact the bacterial population belongs to a virulent strain, that is, the amoebae are infected by bacteria and they die. The authors take this into account by assuming that amoebae are attacked by bacteria following a Holling type II function, with handling time τ and killing rate γ. The nonlocal term in (1) describes the fact that amoebae behave like a sole organism when food supply is low, in order to redistribute the food among all cells; and δ is the growth rate of amoebae.

To study (1), it is convenient to explore the behavior of each equation. Motivated by this, we will analyze in this paper the following nonlocal equation:

$$\begin{cases} \mathcal{L}v = \lambda v \dfrac{\int_\Omega A(x)v(x)\,\mathrm{d}x}{\int_\Omega v(x)\,\mathrm{d}x} - g(x,v) & \text{in} \quad \Omega, \\ v = 0 & \text{on} \quad \partial\Omega, \end{cases} \tag{2}$$

where Ω is a bounded and regular domain of $\mathbb{R}^N$, $A \in C(\overline{\Omega})$ is a nonnegative and nontrivial function. The operator $\mathcal{L}$ is a second order uniformly elliptic operator of the form:

$$\mathcal{L} = -\sum_{i,j=1}^{N} a_{ij}(x)\partial_i\partial_j + \sum_{j=1}^{N} b_j(x)\partial_j + c(x), \tag{3}$$

with

$$a_{ij} \in C(\overline{\Omega}), \quad a_{ij} = a_{ji}, \quad b_j, c \in L^\infty(\Omega), \quad i,j \in \{1,\dots,N\}. \tag{4}$$

Let us briefly recall that $\mathcal{L}$ is a uniformly elliptic operator when there exists a constant $\alpha > 0$ such that, for each $x \in \overline{\Omega}$ and $\xi = (\xi_1,\dots,\xi_N) \in \mathbb{R}^N$, we have

$$\sum_{i,j=1}^{N} a_{ij}(x)\xi_i\xi_j \geq \alpha\, |\xi|^2.$$

In (2) the real number λ is a bifurcation parameter and $g\colon \overline{\Omega} \times \mathbb{R} \to \mathbb{R}$ is a continuous function satisfying $g(x,0) \equiv 0$ and one of the assumptions:

$$\lim_{t\to 0^+} \frac{g(x,t)}{t} = 0, \text{ uniformly in } \Omega, \tag{5}$$

or

$$\lim_{t\to 0^+} \frac{g(x,t)}{t} = g_0(x), \text{ uniformly in } \Omega, \tag{6}$$

where $g_0\colon \overline{\Omega} \to \mathbb{R}$ is a bounded, nonnegative and nontrivial function.

When the function A is constant, (2) transforms into the local equation:

$$\begin{cases} \mathcal{L}v = \lambda A v - g(x,v) & \text{in} \quad \Omega, \\ v = 0 & \text{on} \quad \partial\Omega. \end{cases}$$

The bifurcation theory was applied to these types of problems by several authors, with different assumptions on g, including (5) and (6), see for instance [2,3,10] and references in those papers.