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Complex dynamics of a diffusive predator–prey model with strong Allee effect and threshold harvesting

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ABSTRACT

In this paper, complex dynamics of a diffusive predator–prey model is investigated, where the prey is subject to strong Allee effect and threshold harvesting. The existence and stability of nonnegative constant steady state solutions are discussed. The existence and nonexistence of nonconstant positive steady state solutions are analyzed to identify the ranges of parameters of pattern formation. Spatially homogeneous and nonhomogeneous Hopf bifurcation and discontinuous Hopf bifurcation are proved. These results show that the introduction of strong Allee effect and threshold harvesting increases the system spatiotemporal complexity. Finally, numerical simulations are presented to validate the theoretical results.

1. Introduction

In the early twentieth century [2], the Allee effect had been formalised and initiated by the work of the biologist W.C. Allee. The classical logistic growth model shows that a population has a slow growth at the beginning, then follows a rapid acceleration and finally reaches a phase of stationary growth. Allee argued that such growth may exist, but there may be also a growth so that the species become extinct in a small population density. The Allee effect may owe to all kinds of mechanisms operating in small population sizes, including reduced foraging efficiency in social animals, difficulties in finding mates, lessened defenses against predators, and reduced reproductive success in cooperative breeders [3,7,12,27]. Some experimental evidences of Allee effects have been reported in many populations including insects [24], plants [16,17], marine invertebrates [40], bacteria [39], mammals and birds [11,13]. Allee effects are mainly classified into two types: strong and weak [8,11,30,43]. A strong Allee effect means that the growth is negative when the

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size of the population is below certain threshold value, while a weak one refers to the phenomenon that the population below a threshold has a positive growth.

The difficulty of finding mates among the species at low population density is one of the main causes for the Allee effect. As pointed out in [5], accumulating evidence indicates that two or more Allee effects generating mechanisms can act simultaneously upon a single population.

A predator–prey system is a basic dynamical model to describe the interaction between species [6,9]. The following ODE system with the Allee effect and linear functional response:

$$\begin{cases} \dfrac{\mathrm{d}u}{\mathrm{d}t} = \dfrac{ru}{u+c}\left(1-\dfrac{u}{K}\right)(u-A) - muv, \\ \dfrac{\mathrm{d}v}{\mathrm{d}t} = -\eta v + muv \end{cases} \tag{1}$$

was considered in [8,42], where r is the prey intrinsic growth rate, K is the carrying capacity, A is the Allee effect threshold, c is an auxiliary parameter ($c > 0$, see [7]), m is a scaling of the predator–prey encounter rate, and η is the predator death rate. Here the Allee effect allows the modelling of two Allee effects [5] and $0 < A < K$ represents the strong Allee effect.

The inclusion of spatial dispersal, either physical diffusion or dispersal by random walks, makes the dynamics of the organisms even more complicated. The corresponding spatial dispersal versions of predator–prey systems with strong Allee effect have been gained a lot of attentions, and the spatiotemporal pattern formation in such systems was studied. For instance, the influence of the Allee effect may change the system dynamics significantly resulting in new exotic regimes such as standing chaotic patches and travelling population pulses (see [28,29,35,36,43]). Wang et al. [42,44], Ni and Wang [30], Yang and Zhong [46], Rao and Kang [37] discussed the existence and noexistence of nonconstant steady state solutions, analyzed stationary patterns induced by diffusions for reaction–diffusion functional response predator–prey systems with strong Allee effect in the prey, and showed that the impact of the Allee effect increases the system spatiotemporal complexity. Peng and Zhang [34] considered Turing instability and pattern induced by cross-diffusion in a predator–prey system with Allee effect. Cui et al. [14] studied a diffusive predator–prey system with strong Allee effect and a protection zone, and claimed that the overexploitation phenomenon can be avoided if the Allee effect threshold is low and the protection zone is large.

Most predator–prey models with harvesting, such as fishery harvesting [41] and pest management, only considered constant or linear harvesting functions [4,20–23,45], which were not very realistic argued by Rebaza [38].

In this paper, we assume that the harvesting rate is proportional to the predator population size until it reaches a threshold value due to limited facilities of harvesting or resource protection, then keep at a constant h once the population density arrives its threshold value T. Taking into account the inhomogeneous distribution of the predators and their preys in different spatial locations within a fixed bounded domain $\Omega \subset \mathbb{R}^n$ with smooth boundary at any given time, and the natural tendency of each species to diffuse toward the areas of smaller population concentration, based on system (1), we consider the following reaction–diffusion system with strong Allee effect and threshold harvesting.

$$\begin{cases} \dfrac{\partial u}{\partial t} = d_1\Delta u + \dfrac{ru}{u+c}(1-\dfrac{u}{K})(u-A) - muv - H(u), x \in \Omega, t > 0, \\ \dfrac{\partial v}{\partial t} = d_2\Delta v + v\left(mu - \eta\right), x \in \Omega, t > 0, \\ \dfrac{\partial u}{\partial v} = \dfrac{\partial v}{\partial v} = 0, x \in \partial\Omega, t \geq 0, \\ u(x,0) = u_0(x) \geq 0, v(x,0) = v_0(x) \geq 0, x \in \bar{\Omega}, \end{cases} \tag{2}$$