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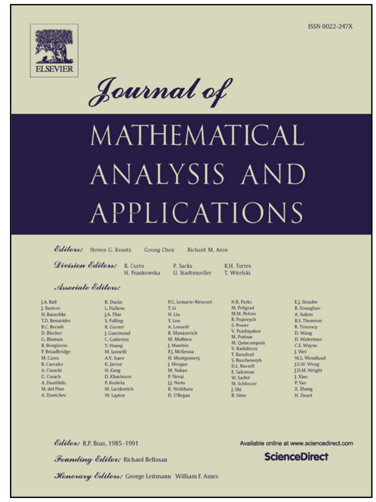

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ON THE SUM OF TWO SUBNORMAL KERNELS

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Abstract. We show, by means of a class of examples, that if K_1 and K_2 are two positive definite kernels on the unit disc such that the multiplication by the coordinate function on the corresponding reproducing kernel Hilbert space is subnormal, then the multiplication operator on the Hilbert space determined by their sum $K_1 + K_2$ need not be subnormal. This settles a recent conjecture of Gregory T. Adams, Nathan S. Feldman and Paul J. McGuire in the negative. We also discuss some cases for which the answer is affirmative.

1. Introduction

Let $\mathcal{H}$ be a complex separable Hilbert space. Let $B(\mathcal{H})$ denote the Banach algebra of bounded linear operators on $\mathcal{H}$. Recall that an operator T in $B(\mathcal{H})$ is said to be *subnormal* if there exists a Hilbert space $\mathcal{K} \supset \mathcal{H}$ and a normal operator N in $B(\mathcal{K})$ such that $N(\mathcal{H}) \subset \mathcal{H}$ and $N|_{\mathcal{H}} = T$. For the basic theory of subnormal operators, we refer to [12].

Completely hyperexpansive operators were introduced in [6]. An operator $T \in B(\mathcal{H})$ is said to be completely hyperexpansive if

$$\sum_{j=0}^{n} (-1)^j \binom{n}{j} T^{*j} T^j \leq 0 \qquad (n \geq 1).$$

The theory of subnormal and completely hyperexpansive operators are closely related with the theory *completely monotone* and *completely alternating* sequences (cf. [5], [6]).

Let $\mathbb{Z}_+$ denote the set of non-negative integers. A sequence $\{a_k\}_{k\in\mathbb{Z}_+}$ of positive real numbers is said to be a completely monotone if

$$\sum_{j=0}^{n} (-1)^j \binom{n}{j} a_{m+j} \geq 0 \qquad (m, n \geq 0).$$

It is well-known that a sequence $\{a_k\}_{k\in\mathbb{Z}_+}$ is completely monotone if and only if the sequence $\{a_k\}_{k\in\mathbb{Z}_+}$ is a Hausdorff moment sequence, that is, there exists a positive measure ν supported in $[0,1]$ such that $a_k = \int_{[0,1]} x^k d\nu(x)$ for all $k \in \mathbb{Z}_+$ (cf. [9]).

Similarly, a sequence $\{a_k\}_{k\in\mathbb{Z}_+}$ of positive real numbers is said to be completely alternating if

$$\sum_{j=0}^{n} (-1)^j \binom{n}{j} a_{m+j} \leq 0 \qquad (m \geq 0, n \geq 1).$$

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