

Accepted Manuscript

Exact rates of convergence in some martingale central limit theorems

Xiequan Fan

PII: S0022-247X(18)30798-4
DOI: <https://doi.org/10.1016/j.jmaa.2018.09.049>
Reference: YJMAA 22569

To appear in: *Journal of Mathematical Analysis and Applications*

Received date: 30 May 2017

Please cite this article in press as: X. Fan, Exact rates of convergence in some martingale central limit theorems, *J. Math. Anal. Appl.* (2018), <https://doi.org/10.1016/j.jmaa.2018.09.049>

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.



Exact rates of convergence in some martingale central limit theorems

Xiequan Fan

*Center for Applied Mathematics, Tianjin University, 300072 Tianjin, China;**Regularity Team, Inria, France***Abstract**

Renz [14], Ouchti [12], El Machkouri and Ouchti [4] and Mourrat [13] have established some tight bounds on the rate of convergence in the central limit theorem for martingales. In the present paper a modification of the methods, developed by Bolthausen [1] and Grama and Haeusler [7], is applied for obtaining exact rates of convergence in the central limit theorem for martingales with differences having conditional moments of order $2 + \rho$, $\rho > 0$. Our results generalise and strengthen the bounds mentioned above.

Keywords: Martingales, Central limit theorem, Berry-Esseen bounds

2000 MSC: Primary 60G42; 60F05; Secondary 60E15

1. Introduction

Assume that we are given a sequence of martingale differences $(\xi_i, \mathcal{F}_i)_{i=0, \dots, n}$, defined on some probability space $(\Omega, \mathcal{F}, \mathbf{P})$, where $\xi_0 = 0$ and $\{\emptyset, \Omega\} = \mathcal{F}_0 \subseteq \dots \subseteq \mathcal{F}_n \subseteq \mathcal{F}$ are increasing σ -fields. Set

$$X_0 = 0, \quad X_k = \sum_{i=1}^k \xi_i, \quad k = 1, \dots, n. \quad (1)$$

Then $X = (X_k, \mathcal{F}_k)_{k=0, \dots, n}$ is a martingale. Let $\langle X \rangle$ be its conditional variance:

$$\langle X \rangle_0 = 0, \quad \langle X \rangle_k = \sum_{i=1}^k \mathbf{E}[\xi_i^2 | \mathcal{F}_{i-1}], \quad k = 1, \dots, n. \quad (2)$$

Define

$$D(X_n) = \sup_{x \in \mathbf{R}} \left| \mathbf{P}(X_n \leq x) - \Phi(x) \right|,$$

where $\Phi(x)$ is the distribution function of the standard normal random variable. Denote by $\xrightarrow{\mathbf{P}}$ convergence in probability. According to the basic results of martingale central limit theory (see the

*Corresponding author.

E-mail: fanxiequan@hotmail.com (X. Fan).

Download English Version:

<https://daneshyari.com/en/article/11010187>

Download Persian Version:

<https://daneshyari.com/article/11010187>

[Daneshyari.com](https://daneshyari.com)