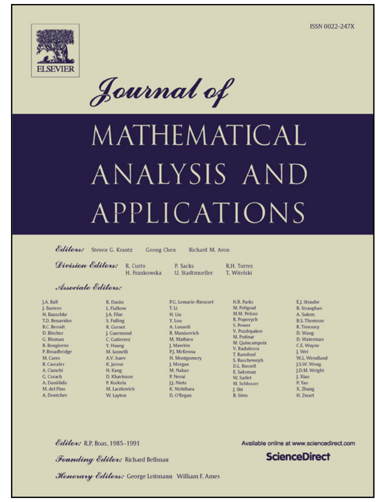
Accepted Manuscript



Exact rates of convergence in some martingale central limit theorems

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PII: S0022-247X(18)30798-4
DOI: https://doi.org/10.1016/j.jmaa.2018.09.049
Reference: YJMAA 22569

To appear in: *Journal of Mathematical Analysis and Applications*

Received date: 30 May 2017

Please cite this article in press as: X. Fan, Exact rates of convergence in some martingale central limit theorems, *J. Math. Anal. Appl.* (2018), https://doi.org/10.1016/j.jmaa.2018.09.049

Exact rates of convergence in some martingale central limit theorems

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Abstract

Renz [14], Ouchti [12], El Machkouri and Ouchti [4] and Mourrat [13] have established some tight bounds on the rate of convergence in the central limit theorem for martingales. In the present paper a modification of the methods, developed by Bolthausen [1] and Grama and Haeusler [7], is applied for obtaining exact rates of convergence in the central limit theorem for martingales with differences having conditional moments of order $2+\rho, \rho>0$. Our results generalise and strengthen the bounds mentioned above.

Keywords: Martingales, Central limit theorem, Berry-Esseen bounds

2000 MSC: Primary 60G42; 60F05; Secondary 60E15

1. Introduction

Assume that we are given a sequence of martingale differences $(\xi_i, \mathcal{F}_i)_{i=0,...,n}$, defined on some probability space $(\Omega, \mathcal{F}, \mathbf{P})$, where $\xi_0 = 0$ and $\{\emptyset, \Omega\} = \mathcal{F}_0 \subseteq ... \subseteq \mathcal{F}_n \subseteq \mathcal{F}$ are increasing σ-fields. Set

$$X_0 = 0, \qquad X_k = \sum_{i=1}^{k} \xi_i, \quad k = 1, ..., n. \tag{1}$$

Then $X = (X_k, \mathcal{F}_k)_{k=0,...,n}$ is a martingale. Let $\langle X \rangle$ be its conditional variance:

$$\langle X \rangle_0 = 0, \qquad \langle X \rangle_k = \sum_{i=1}^{k} \mathbf{E}[\xi_i^2 | \mathcal{F}_{i-1}], \quad k = 1, ..., n. \tag{2}$$

Define

$$D(X_n) = \sup_{x \in \mathbf{R}} \Big| \mathbf{P}(X_n \le x) - \Phi(x) \Big|,$$

where $\Phi(x)$ is the distribution function of the standard normal random variable. Denote by $\xrightarrow{\mathbf{P}}$ convergence in probability. According to the basic results of martingale central limit theory (see the

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