

Fixed Points and Dynamics of Two-Parameter Family of Hyperbolic Cosine like Functions

Madhusudan Bera , M. Guru Prem Prasad *

*Department of Mathematics, Indian Institute of Technology Guwahati,
PIN 781039, INDIA*

Abstract

In this paper, the two-parameter family $\mathcal{F}_b \equiv \{f_{\lambda,\mu}(z) = \lambda b^z + \mu b^{-z} \text{ for } z \in \mathbb{C} : \lambda \geq \mu > 0\}$ of transcendental entire functions is considered. By investigating on the existence and nature of the fixed points of $f_{\lambda,\mu}$, the dynamics of functions $f_{\lambda,\mu} \in \mathcal{F}_b$ is studied. It is shown that the Julia set of $f_{\lambda,\mu}$ is nowhere dense and not locally connected whenever parameters λ and μ satisfy $\sqrt{1 + 4\lambda\mu(\ln b)^2} \leq \ln \left(\frac{1 + \sqrt{1 + 4\lambda\mu(\ln b)^2}}{2\lambda \ln b} \right)$ and the Julia set is the whole of extended complex plane for $\sqrt{1 + 4\lambda\mu(\ln b)^2} > \ln \left(\frac{1 + \sqrt{1 + 4\lambda\mu(\ln b)^2}}{2\lambda \ln b} \right)$.

Key words: Fatou sets, Julia sets, Transcendental entire functions.

1 Introduction

The iteration theory of complex holomorphic functions is mainly developed by Fatou and Julia in early 20th century. In iteration theory, the dynamics of entire and meromorphic functions has been studied extensively. In the end of 20th and beginning of 21st century, dynamics of one-parameter family of functions such as $\lambda \exp z$, $i\lambda \cos z$, $\lambda \sinh(z)/z$, $\lambda \tanh(e^z)$, $\lambda(b^z - 1)/z$ have been studied by many researchers. Devaney vastly studied the one-parameter family $\mathcal{E} \equiv \{E_\lambda(z) = \lambda \exp z : \lambda > 0\}$. Devaney [4] and Devaney and Durkin [6] proved that for $0 < \lambda < (1/e)$, the Julia set $J(E_\lambda)$ is a nowhere dense subset entirely contained in the right half plane. When the parameter λ crosses

* Corresponding author.

Email addresses: m.bera@iitg.ac.in (Madhusudan Bera), mgpp@iitg.ac.in (M. Guru Prem Prasad).

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