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Global strong solutions of the tropical climate model with temperature-dependent diffusion on the barotropic mode

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ABSTRACT

This paper is concerned with the tropical model with temperature-dependent diffusion on the barotropic mode in $\mathbb{R}^2 \times \mathbb{R}_+$. We prove the existence of the global strong solution under the initial data $\|u_0\|^2_{H^s} + \|v_0\|^2_{H^s} + \|\theta_0\|^2_{H^s}$ with $s > 1$ is suitably small.

1. Introduction

By using a Galerkin truncation to the hydrostatic Boussinesq equations, Frierson, Majda and Pauluis [1] derived the original version of tropical climate system in $\mathbb{R}^2$:

$$\begin{cases} u_t + u \cdot \nabla u + \nabla p = -\mathrm{div}(v \otimes v), \\ v_t + u \cdot \nabla v + v \cdot \nabla u = \nabla \theta, \\ \theta_t + u \cdot \nabla \theta = \mathrm{div} v, \\ \mathrm{div}\, u = 0. \end{cases} \tag{1.1}$$

The unknowns are the vector fields $u = (u_1, u_2), v = (v_1, v_2)$ and the scalar functions θ and p. Here, u and v are the barotropic mode and the first baroclinic mode of the velocity, respectively. θ and p denote the temperature and the pressure, respectively.

Several works focused on the mathematical problem by adding some diffusions in the original version. Li and Titi [2] established the global well-posedness of strong solutions for the system with Laplace diffusions

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(adding $-\Delta u$ and $-\Delta v$ in (1.1)$_{1,2}$). They introduced a new pseudo baroclinic velocity

$$\omega = v - \nabla(-\Delta)^{-1}\theta$$

to overcome the difficulty caused by the absence of the diffusivity in the temperature equation. Some small initial data results were studied by Wan in [3] and Ma–Wan in [4]. Ye [5] obtained global regularity for a class of 2D tropical climate model (adding $-\Delta v$, $-\Delta\theta$ in (1.2) and (1.3)) with $\alpha > 0$. Dong et al. [6] obtained global regularity for the 2D tropical climate model with fractional dissipation(adding $\Lambda^{2\beta}v$ in (1.2)) when $\alpha + \beta = 2$, $1 < \beta \le \frac{3}{2}$. Ma, Jiang and Wan [7] established the local well-posedness of strong solutions to this generalized model (adding $\Lambda^{2\beta}v$ in (1.2)). In [8], global regularity for the tropical climate model with fractional diffusion on barotropic mode (adding $\Lambda^{2\alpha}u$, $\alpha \ge \frac{5}{2}$) was studied.

Motivated by [9–11], we consider the following the tropical climate system with temperature-dependent viscosity on the barotropic mode:

$$\begin{cases} u_t + u\cdot\nabla u + \nabla p - \operatorname{div}(\mu(\theta)\nabla u) + \alpha u = -\operatorname{div}(v\otimes v), \\ v_t + u\cdot\nabla v + v\cdot\nabla u + \alpha v = \nabla\theta, \\ \theta_t + u\cdot\nabla\theta = \operatorname{div} v, \\ \operatorname{div} u = 0. \end{cases} \tag{1.2}$$

α is the nonnegative parameter, $\mu(\theta)$ is the viscosity coefficient of the barotropic mode and concerned with temperature, and satisfies

$$\mu(\theta) \in C^s([0,+\infty)), \quad (s>1) \quad \textit{and} \quad 0 < \underline{\mu} \le \mu(\theta) \le \bar{\mu} < \infty. \tag{1.3}$$

Theorem 1.1. *Assume that (1.3) holds and the initial data (u_0, v_0, θ_0) satisfy*

$$(u_0, v_0, \theta_0) \in H^s, \quad s > 1. \tag{1.4}$$

Then there exists a positive ε_0 depending $\mu(\theta)$ such that if

$$\|u_0\|_{H^s}^2 + \|v_0\|_{H^s}^2 + \|\theta_0\|_{H^s}^2 \le \varepsilon_0, \tag{1.5}$$

the Cauchy problem (1.2) (1.4) has a unique global strong solution (u, v, θ) in $\mathbb{R}^2 \times (0,\infty)$ satisfying

$$\sup_{0\le t\le T}(\|v\|_{H^s}^2 + \|\theta\|_{H^s}^2 + \|u\|_{H^s}^2) + \int_0^T \left(\|v\|_{H^s}^2 + \|u\|_{H^{s+1}}^2\right) dt \le C. \tag{1.6}$$

Theorem 1.1 will be proved by combining the local existence theorem and Proposition 1.1. The local existence can be established by the contraction map principle. The proof is lengthy but standard, and thus it is omitted here. The uniqueness of the strong solutions can be proved in a standard manner. So, we only need to prove the following proposition.

Proposition 1.1. *Under the conditions of Theorem 1.1, there exists some positive constant ε_1 such that if (u, v, θ) is a strong solution of (1.2) (1.4) on $\mathbb{R}^2 \times (0, T]$ satisfying*

$$\sup_{0\le t\le T}(\|u\|_{H^s}^2 + \|v\|_{H^s}^2 + \|\theta\|_{H^s}^2) \le 2\varepsilon_1 \tag{1.7}$$

the following estimates hold

$$\sup_{0\le t\le T}(\|u\|_{H^s}^2 + \|v\|_{H^s}^2 + \|\theta\|_{H^s}^2) \le \varepsilon_1. \tag{1.8}$$

In this paper, we use $\hat{f}$ to denote the Fourier transform $\hat{f}(\xi) = \int_{\mathbb{R}^n} f(x)e^{-ix\xi}dx$. $\Lambda^s = (-\Delta)^{s/2}$ is defined in terms of Fourier transform by $\widehat{\Lambda^s f}(\xi) = |\xi|^s\hat{f}(\xi)$. $\dot{H}^s(\mathbb{R}^n)$ and $H^s(\mathbb{R}^n)$ denote the homogeneous Sobolev spaces $\|u\|_{\dot{H}^s} = \|\Lambda^s u\|_{L^2}$ and nonhomogeneous Sobolev spaces $\|u\|_{H^s} \triangleq \|u\|_{L^2} + \|\Lambda^s u\|_{L^2}$.