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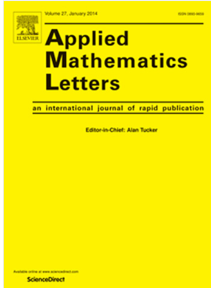

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Non-singular Green's functions for the unbounded Poisson equation in one, two and three dimensions

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Abstract

In this paper, we derive the non-singular Green's functions for the unbounded Poisson equation in one, two and three dimensions using a spectral cut-off function approach to impose a minimum length scale in the homogeneous solution. The resulting non-singular Green's functions are relevant to applications which are restricted to a minimum resolved length scale (e.g. a mesh size h) and thus cannot handle the singular Green's function of the continuous Poisson equation. We furthermore derive the gradient vector of the non-singular Green's function, as this is useful in applications where the Poisson equation represents potential functions of a vector field.

Keywords: Partial differential equations, Poisson equation, Green's function, unbounded domain

1. Introduction

Green's functions are the preferred method for solving linear differential equations in an unbounded domain, i.e. with free-space boundary conditions. The Green's function represents a homogeneous solution which is derived analytically, and then used to obtain the particular solution by a convolution with the right-hand-side field of the Poisson equation.

The analytical Green's function of a continuous smooth field is singular at its origin. Applied in discretized numerical calculations, the singularity of the Green's function evidently causes a number of difficulties. In order to amend this, smoothing regularization techniques have been applied (e.g. [1, 2, 3]) which introduce a continuous and smooth field distribution around the discrete points and thus avoid the singularity of the Green's function. However, most regularization methods that have been applied are based on functions that only conserve a finite number of field moments, and are thus only accurate up to a finite order of convergence rate.

Vico et al. [4] derived non-singular Fourier transforms of the Green's function in two and three dimensions by imposing an isotropic maximum length scale of the integrated domain in real space. This method

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