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Discrete Optimization

Distributionally robust scheduling on parallel machines under moment uncertainty

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ABSTRACT

This paper investigates a distributionally robust scheduling problem on identical parallel machines, where job processing times are stochastic without any exact distributional form. Based on a distributional set specified by the support and estimated moments information, we present a min-max distributionally robust model, which minimizes the worst-case expected total flow time out of all probability distributions in this set. Our model doesn't require exact probability distributions which are the basis for many stochastic programming models, and utilizes more information compared to the interval-based robust optimization models. Although this problem originates from the manufacturing environment, it can be applied to many other fields when the machines and jobs are endowed with different meanings. By optimizing the inner maximization subproblem, the min-max formulation is reduced to an integer second-order cone program. We propose an exact algorithm to solve this problem via exploring all the solutions that satisfy the necessary optimality conditions. Computational experiments demonstrate the high efficiency of this algorithm since problem instances with 100 jobs are optimized in a few seconds. In addition, simulation results convincingly show that the proposed distributionally robust model can hedge against the bias of estimated moments and enhance the robustness of production systems.

1. Introduction

The parallel machine scheduling problem (PMSP) considers a set of jobs to be processed on several identical machines over a given time period. Each job has to be processed on one of the machines, and no machine can process more than one job at the same time. A feasible schedule contains the allocation of machines and the sequence of jobs on each machine. The aim of PMSP is to find a schedule, which optimizes one or more performance criteria, among all feasible ones. As a decision-making process, it plays an important role in many manufacturing and service industries (Pinedo, 2012). Although PMSP is formulated in the manufacturing environment, this model is suitable for many other fields (Mokotoff, 2004), such as social science (Liao, Van Delft, & Vial, 2013; Mak, Rong, & Zhang, 2014b) and computer science (Kim, Park, Cui, Kim, & Gruver, 2009). For example in outpatient clinics, the doctors and patients can be regarded as the machines and jobs, respectively. The turnaround time elapsed from a patient's coming to its leaving, which is the key performance measure of appointment systems, is related to the flow time of a job in PMSP. The deterministic PMSP has been extensively studied in the past decades (Cheng & Sin, 1990; Edis, Oguz, & Ozkarahan, 2013; Valada & Ruiz, 2011). However, the job processing times are always uncertain because of the personalized customization, the unknown operation time for a new patient, or some other uncertain factors. Under this circumstance, the 'optimal' schedules obtained by deterministic models may turn out to be poor in practice (Daniels & Kouvelis, 1995), which motivates us to study scheduling problems with uncertainties.

Popular approaches applied to deal with uncertainties in scheduling are stochastic program (SP) and robust optimization (RO), both of which have inherent drawbacks. SP models regard the uncertain parameters as random variables with known distributions (Pinedo, 1983; Skutella & Uetz, 2005; Van Der Heyden, 1981; Weber, Varaiya, & Walrand, 1986), but the exact distributions are hard to obtain due to limited information in practice. While RO models (Daniels & Kouvelis, 1995; Tadayon & Smith, 2015; Yang & Yu, 2002) restrict the uncertain parameters to given uncertainty sets without any distributional assumption, which fails to make full use of the information that may be obtained via historical data. To overcome these shortcomings, we propose a distributionally robust (DR) formulation for PMSP, where uncertain parameters are regarded as random variables subject to an ambiguous distribution limited in a distributional set. Besides the support of uncertain

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parameters, the mean and covariance matrix are also involved in the formulation, which makes the DR models contain more information than the RO ones.

In this paper, we study an identical PMSP with uncertain job processing times in DR framework. The objective functions of PMSP are generally related to the completion times or the due dates of different jobs, such as the maximum completion time, i.e., makespan (Koulamas & Kyparisis, 2009; Mokotoff, 2004), the total flow time (Li & Yang, 2009; Liaw, 2016; Weng, Lu, & Ren, 2001), the total/maximum tardiness (Bilge, Kiraç, Kurtulan, & Pekgün, 2004; Lee, 2017) and the number of tardy jobs (Ho & Chang, 1995). Because the flow time of each job means the time elapsed from the job's release to its completion, the total flow time (TFT) indicates the total holding or inventory costs incurred by a schedule. Among different metrics, TFT is widely accepted as a good measure for the overall quality of production. Furthermore, it is linear in job processing times, which is a favorable property for our model transformation. Therefore, we choose TFT as the performance measure and develop the DR formulation based on a distributional set specified by the support and first two moments information of job processing times. To consider the uncertainties in moments, our distributional set restricts the mean vector in an ellipsoid and the covariance matrix in a positive semidefinite cone. The original formulation is a two-stage min-max problem, which aims at finding a robust schedule with minimal expected TFT under the worst-case distribution. By solving the inner maximization subproblem, the min-max formulation is finally reduced to an integer second-order cone program (I-SOCP).

In general, I-SOCP can be solved by some commercial solvers, such as IBM® ILOG® CPLEX and Mosek, via branch-and-bound based methods. However, these methods are time-consuming when the size of problem instances is large. In production environments, scheduling decisions should be made for each work shift with the time-frame usually being days or hours. However, rescheduling is often required to modify schedules due to uncertainties in the production process, and has to be completed within a much shorter computational time limit. To solve our I-SOCP with higher efficiency, we propose an exact algorithm based on its structural features, which can find out all the solutions satisfying the necessary optimality condition (i.e., NOC-points) and reserve the best one as the optimal solution. This algorithm is intuitively named as NOC-point search algorithm (NPSA). The optimality of NPSA is rigorously proved, and its efficiency is demonstrated by solving large problem instances with up to 100 jobs in a few seconds.

The main contributions in this paper are summarized as follows:

1. We propose a DR optimization approach to model the PMSP with uncertain parameters, which avoids the shortcomings of existing SP and RO models.
2. We consider the uncertainty of estimated moments in our DR formulation, which hedges against the bias caused by insufficient historical data in practice.
3. We reduce the original min-max DR formulation to an I-SOCP problem.
4. We design an exact algorithm for our I-SOCP problem, which is efficient and easy to implement. The design ideas can be extended to solve other I-SOCP problems with similar structure in different applications.

The rest of this paper is organized as follows. Related literature is reviewed in Section 2, and the formulation of PMSP as well as its DR counterpart are introduced in Section 3. Next in Section 4, we propose the exact algorithm NPSA and provide its optimality proof. In Section 5, we illustrate the empirical performance of NPSA, and analyze the robustness of our DR-PMSP model. Finally in Section 6, we conclude this paper with a few final remarks.

2. Related literature

A common approach to deal with uncertainties in production scheduling problems is stochastic programming (SP). These SP models regard the uncertain parameters as random variables with known distributions, and usually aim at optimizing the expected performance. For example, Pinedo (1983) developed four SP models, where the job processing times are assumed to be exponentially distributed. Three of them are single machine scheduling problems, and the other one is a PMSP with the expected number of late jobs being the performance measure. Van Der Heyden (1981), Weber et al. (1986) and Skutella and Uetz (2005) also considered this kind of stochastic PMSP models, but their objective functions are the expected makespan or completion time.

SP models can gain good expected performance in the long run, while performance fluctuations are rarely considered. To hedge against the risk of achieving substandard performance, a so-called 'β-robust' scheduling model was developed. In these models, the distributions of random parameters are also known in advance, but the aim is to minimize the likelihood that some performance measure exceeds a given threshold. Daniels and Carrillo (1997) and Wu and Zhou (2008) proposed 'β-robust' scheduling models for a single machine problem with normally distributed job processing times. Ranjbar, Davari, and Leus (2012) and Pishevar and Tavakkoi-Moghaddam (2014) extended these studies to PMSP with the same setting of job processing times. Although 'β-robust' scheduling models consider the risk-averse attitude of decision makers, exact distributions of random parameters are also required like SP models. This is a common drawback of the two kinds of models when the exact distribution is hard to obtain.

Another approach used to deal with uncertainties in scheduling is the fuzzy set theory or possibility theory. Among existing literature, there are mainly three different ways to utilize the fuzzy theories. The first one fuzzifies directly the classical scheduling rules or the deterministic scheduling models (Balin, 2011; Mok, Kwong, & Wong, 2007; Özelkan & Duckstein, 1999), where the uncertain parameters are usually described as fuzzy numbers, and appropriate fuzzy ranking and defuzzification methods are to be determined. The second one does not represent uncertainties by fuzzy sets, but uses fuzzy logic to design decision support systems (Petrovic & Duenas, 2006) or combine different objective functions (Raja, Arumugam, & Selladurai, 2008). The third type is based on the possibility, necessity and credibility measure of a fuzzy variable, which follows a similar modeling philosophy in the stochastic scheduling environment (Peng & Liu, 2004). Fuzzy theory avoids using exact probability distributions of uncertain parameters and enables the use of fuzzy rules in heuristic algorithms. However, these fuzzy models might be effective only for the chosen fuzzy sets, fuzzy rules or the method of fuzzy simulations. In addition, the stability of production systems gained by these models is rarely analyzed in the literature.

To enhance the robustness and stability of production schedules, robust optimization (RO) was applied to deal with uncertain production scheduling problems. RO models (Ben-Tal, El Ghaoui, & Nemirovski, 2009; Gabrel, Murat, & Thiele, 2014) optimize system performance in the worst-case scenario, where uncertain parameters are regarded as variables limited in given uncertainty sets. Since RO approach was first introduced by Daniels and Kouvelis (1995) to deal with the uncertain single machine scheduling problem, plenty of research on robust scheduling has sprung out (Aloulou & Della Croce, 2008; de Farias, Zhao, & Zhao, 2010; Lu, Lin, & Ying, 2014; Tadayon & Smith, 2015; Yang & Yu, 2002). Xu, Cui, Lin, and Qian (2013) considered a robust identical PMSP