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A chirp excitation for focussing flexural waves

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ABSTRACT

In this paper, the dispersive nature of flexural waves is exploited to generate a shock response at an arbitrary location on a waveguide. The input waveform is an up-chirp whose instantaneous frequency is chosen to ensure synchronous arrival at an arbitrary focal point. An analytical expression is derived for the required chirp waveform as a function of bandwidth and focal point location given prior knowledge of the dispersion relation.

The principle is illustrated for an analytical model of a uniform beam. Simulated results show that it is possible, in theory, to achieve peak responses that are at least an order of magnitude larger than steady state response due to harmonic excitation. Further, the peak response increases with approximately the square root of distance from the point of excitation when damping is negligible. Velocity, acceleration, normal strain and shear stress exhibit qualitatively similar results which differ quantitatively owing to their different frequency responses with respect to the input.

A single degree-of-freedom model of an electrodynamic shaker is coupled to the analytical beam model in order to predict peak mechanical responses per peak input voltage of the chirp waveform. The coupled electromechanical model is then validated experimentally through both frequency response and transient measurements. The technique is potentially applicable to situations where a large and reasonably localised transient response is required on a beam or plate-like structure using minimal instrumentation.

1. Introduction

Dispersive waves are so-called because their group velocity is frequency dependent, causing the waveform of a broadband disturbance such as a pulse to spread out as it propagates along a waveguide. This phenomenon is often undesirable, for example in ultrasonic non-destructive evaluation where overlapping wave packets scattered from different features hinder localisation of abnormalities [1]. This paper is motivated by a different intent, namely the generation of large shock responses in structures for accretion removal. Potential applications include windows, pipes, ship hulls and aircraft wings. Removal of ice from the latter has been successfully demonstrated previously using steady state rather than shock response, e.g. Ref. [2] but is limited by factors such as actuator authority and integrity. At first sight wave dispersion may also seem unbeneficial for shock based accretion removal since a shock input results in an increasingly distorted response far from the input location. However, it is shown here that by careful choice of input waveform it is possible to compensate for and even exploit dispersion to good effect. The input waveform can be arranged to launch frequency components with the correct delays for them to arrive at a target position simultaneously culminating in a large localised shock. The extended duration of the input waveform also allows far more energy to be input to the system than by an impulse.

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Derivation of the necessary input waveform can be accomplished using time reversal acoustics which is an established technique for compensating for the effects of forward wave propagation [3,4]. In time reversal acoustics, a desired response waveform at a specified point of interest is constructed in a two-step process. In the forward propagation step, a source at the point of interest excites the system using the desired response waveform, and an array of sensors is deployed to acquire a set of resulting responses due to the propagating wave field. In the backward propagation step, the measured responses are re-emitted in reverse through an array of collocated actuators in an attempt to recreate the wave field propagating in reverse time. Assuming invariance of the wave equation to time reversal, which is satisfied only for conservative systems, the response at the original source position can resemble the initial excitation waveform.

Time reversal has been applied to dispersive waves in structures at ultrasonic frequencies by Montaldo et al. [5]. Longitudinal waves were excited at one end of a solid circular waveguide with a time-reversed signal so as to concentrate energy at the other end which is submersed in a fluid. A similar problem was studied by Dion et al. [6] who extracted the required input waveform from measured FRFs instead of time-reversal.

Francoeur and Berry [7] implemented time reversal on an anechoic beam at audible frequencies, principally to identify and locate wave scatterers. They used an array of PVDF sensors and piezoelectric actuators to generate an impulsive acceleration at a remote position that was larger than at the source locations.

The purpose of this paper is to quantify the potential amplification in shock response obtainable by compensating for wave dispersion, by means of a simpler approach to time reversal. A chirp (or rapid swept sine) excitation waveform is formulated given prior knowledge of the dispersion relation of a single wave type. The technique has just a forward propagation step which offers significant benefits when compared to time reversal, particularly for practical implementation:

(i) no sensor array is required (once the dispersion relation is known).
(ii) actuators are not required at the target locations.
(iii) the excitation waveform is determined analytically and is not therefore contaminated by measurement noise.
(iv) the response at the target location is filtered by the system's frequency response only once (but twice with time reversal).

However, the chirp technique accounts for neither wave reflections from boundaries, thereby focussing only incident waves, nor phase delays in the actuation system.

Section 2 of this paper presents the derivation of the chirp waveform for beams and plates in a similar form to that reported in a master's dissertation [8] from which this work originates. In Section 3, a standard frequency domain analytical model of a beam is adopted from which the time response to arbitrary transient inputs can be computed via the inverse Fourier transform. The model is extended to include a coupled electrodynamic shaker for subsequent experimental validation purposes. In Section 4, simulations are presented to quantify the effects of bandwidth, propagation distance and attenuation on achievable peak responses. Section 5 concerns experimental implementation using an electrodynamic shaker, and validation of the coupled beam/shaker model developed in Section 3.

2. The chirp waveform

Rapid sine sweep, or 'chirp' excitation waveforms are used in fields such as modal testing for high fidelity transfer function estimation [9] and in ultrasonic inspection to infer tone burst responses [10]. Often, a linear sweep rate is adopted on account of its flat frequency spectrum. In this paper, the sweep rate is chosen specifically to arrange for synchronous arrival of all frequency components at an arbitrary position which is remote from the input. An analytical expression is derived, based on the analysis of [8], for the case of an infinite uniform beam or plate.

A chirp signal $f(t)$ of duration T has the general form

$$f(t) = \begin{cases} F \sin \phi(t) & 0 \le t \le T \\ 0 & t > T \end{cases} \tag{1}$$

where F is a constant, ϕ is the instantaneous phase, and the instantaneous circular frequency ω is given by

$$\omega(t) = \frac{\mathrm{d}\phi(t)}{\mathrm{d}t} \tag{2}$$

An expression is sought for $\omega(t)$ given prior knowledge of the wave dispersion relation such that all frequency components arrive synchronously at some focal point a distance x_f from the force (or moment) at $x = 0$. The velocity of a wave packet centred at frequency ω, is given by its group velocity,

$$c_g = \left(\frac{\mathrm{d}(\mathrm{Re}\{k\})}{\mathrm{d}\omega} \right)^{-1} \tag{3}$$