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A criterion on asymptotic stability for partially equicontinuous Markov operators

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Abstract

In this paper, we prove a slight, but practically useful, generalisation of a criterion on asymptotic stability for Markov e-chains by T. Szarek, which is based on the so-called lower bound technique, developed by A. Lasota and J. York. Simultaneously, we present an alternative proof of this theorem using an asymptotic coupling method introduced by M. Hairer. Our main result is illustrated by an application to random iterated function systems, which are not contracting on average.

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Keywords: Markov chain; E-property; Asymptotic stability; Invariant measure; Coupling; Iterated function system

1. Introduction

Asymptotic analysis of Markov e-processes has received quite a lot of attention in literature [1,8,12,15,20,21]. The concept of e-process is based on an equicontinuity condition, called e-property, which generalises the notion of non-expansiveness of transition Markov operators with respect to the Fortet–Mourier distance [4,14,18]. The e-property is certainly much more intuitive and easier to verify than, for instance, the strong Feller property or its asymptotic form (introduced in [5]). It turned out to be a valuable tool, for example, in proving the existence of an invariant measure for the Markov semigroup associated with a stochastic heat equation with the so-called impulsive white noise [12], as well as in verifying the weak-* mean ergodicity of the Lagrangian observation process corresponding to a passive tracer model [8].

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Our starting point is the result of T. Szarek [20, Theorem 3.3], which provides sufficient conditions for *asymptotic stability* (see e.g. [1,10,15]) of Markov operators satisfying the e-property. Its idea is based on the lower bound technique [10,12,14], applied for the first time by A. Lasota and J. York in [13]. To be more precise, suppose we are given a time-homogeneous Markov chain, evolving on a separable, complete metric space (X, ρ), with one-step transition law $(x, A) \mapsto P(x, A)$. Then one can define the operator $f \mapsto Pf$, acting on bounded, Borel functions $f : X \to \mathbb{R}$, by

$$Pf = \int_X f(y)\, P(\cdot, dy).$$

If such an operator preserves the continuity then it is called *Feller*. We say that P have the *e-property* if for any bounded, Lipschitz continuous function $f : X \to \mathbb{R}$ and $z \in X$ the family $\{P^n f : n \in \mathbb{N}\}$ is equicontinuous in z, that is

$$(\forall \varepsilon > 0)(\exists \delta > 0)(\forall x \in B(z, \delta),\ n \in \mathbb{N})(|P^n f(x) - P^n f(z)| < \varepsilon).$$

Analogously, if $(x, A) \mapsto Q(x, A)$ is a *substochastic kernel* [15], we can consider the operator $f \mapsto Qf$ and define the e-property for Q. The above-mentioned theorem of T. Szarek says that, if P is Feller, has the e-property, and there exists $z \in X$ such that

$$\inf_{x\in X} \liminf_{n\to\infty} P^n(x, U) > 0 \tag{1.1}$$

for every open neighbourhood U of z, then P is asymptotically stable. Although not considered here, it is worth noting that similar results are also available for continuous time Markov semigroups (see e.g. [21, Corollary 5.4]). In the paper we obtain an analogous theorem (Theorem 5.9) assuming that the e-property is satisfied by a substochastic kernel $(x, A) \mapsto Q(x, A)$ such that $Q \le P$, and $Q^n(\cdot, X)$ does not vanish on some open neighbourhood of z (as $n \to \infty$). Having established this, we further demonstrate that, under an additional assumption, condition (1.1) can be reduced to its local form, which is usually much easier to verify (Corollary 5.12). As an example of the use of this result we consider a class of random iterated function systems (RIFSs) with place-dependent probabilities, which are not necessarily contracting on average (in the sense assumed in [7,14,19,22]) and may contain non-Lipschitz unbounded transformations. We show that Markov operators arising from such dynamical systems are asymptotically stable (Proposition 6.1).

Asymptotic stability of P can be established by proving that P has an invariant distribution π, i.e. $\int_X P(x, \cdot)\pi(dx) = \pi$, and that $|\int_X P^n f\, d\mu - \int_X P^n f\, d\nu| \to 0$ for any Borel probability measures μ, ν and all bounded, Lipschitz continuous functions f. To show that the first claim holds, we essentially follow the proof of [20, Proposition 2.1]. In order to prove the latter we employ the coupling methods introduced by M. Hairer [6] and a general result concerning absorption probabilities (Theorem 3.3), which is based on ideas from [16]. By a *Markovian coupling* of P we mean a Markov chain, taking values in X^2, whose transition law $(x, y, C) \mapsto B(x, y, C)$ satisfies

$$B(x, y, A \times X) = P(x, A) \quad \text{and} \quad B(x, y, X \times A) = P(y, A). \tag{1.2}$$

As will be clarified later (in the proof of Theorem 5.9), one can define a coupling of P with transition law B and a substochastic kernel $(x, y, C) \mapsto S(x, y, C)$ on X^2, satisfying $S \le B$, such that, for any given bounded, Borel function $f : X \to \mathbb{R}$, the equicontinuity of both $\{Q^n f : n \in \mathbb{N}\}$ and $\{Q^n \mathbb{1}_X : n \in \mathbb{N}\}$ at a point z implies that

$$(\forall \varepsilon > 0)(\exists \delta > 0)(\forall x, y \in B(z, \delta),\ n \in \mathbb{N})\left(\left|\int_{X^2} [f(u) - f(v)]\ S^n(x, y, du, dv)\right| < \varepsilon\right).$$

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