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Elastomeric bearing sizing analysis part 2: Flat and cylindrical bearings

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A R T I C L E I N F O

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A B S T R A C T

This analysis covers rigorous development of theoretical equations based on linear elasticity and the assumption of rigid shims (i.e. reinforcements) used in the following bearing sizing computer programs for general three-dimensional loading: Flat (circular) Single Pad, Flat Multilayer, Partial Cylinder (single and multilayer), Complete Cylinder (single and multilayer). Essentially the same methodology based on perturbation theory is used for spherical, flat and cylindrical bearings, and thus the theory was very detailed for only the spherical bearing in Part 1. Part 1 includes equations for predicting beam-column action and buckling for quite general bearing geometries, loads and deformation modes, including tensile, shear and bending. For the two geometries in Part 2 the analysis is less detailed, and differences in the theory, if any, are noted. All equations have been programmed using Mathcad embedded in Excel for ease of use by the designer. The beam-column analysis requires at least two pads, thus necessitating a separate single-pad program. Sample results are shown in this paper for stiffness as a function of shape factor, including comparison with finite element predictions, and for selected displacement, stress, and strain distributions.

1. Introduction

Part 2 continues the use of a perturbation method to solve for the mechanical state of elastomeric bearings. It provides zeroth-order equations that agree well with solutions found by the finite element method for most, if not all, bearing designs. Related publications were discussed in Part 1 (Schapery, 2018a). The developed equations enable the designer to rapidly evaluate a range of bearing designs, select the most promising ones, and then use the finite element method to account for the effects of rubber nonlinearity and shim deformation in the final design stage. Section 2 covers the flat bearing, which is in wide use (Kelly and Konstantinidis, 2011). This analysis is followed by discussion of partial and complete cylindrical bearings. Some predictions of stiffness as a function of shape factor, S, are shown for flat pads and compared with finite element solutions. For both flat and partial cylindrical pads, there are a number of 3-D plots of mechanical variables illustrating the shape of the variable surface without tying it to a specific case.

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2. Flat bearing

The method of analysis is similar to that used for the spherical bearings, although in order to improve accuracy for loading in axial and bending modes with small shape factors (thick pads), a generalization of the perturbation method is used. Only the basic equations, limited discussion of the analyses and selected results are covered in this section.

2.1. Single pad

2.1.1. Cylindrical coordinates; field equations; geometry; boundary conditions

The cylindrical coordinate system (r, z, θ) is shown relative to a Cartesian system (x, y, z) in Fig. 1.

Field equations for the rubber consist of fifteen equations in terms of fifteen dependent variables: six stresses $(\sigma_r, \sigma_z, \sigma_\theta, \tau_{rz}, \tau_{r\theta}, \tau_{z\theta})$, six strains $(\varepsilon_r, \varepsilon_z, \varepsilon_\theta, \gamma_{rz}, \gamma_{r\theta}, \gamma_{z\theta})$ and three displacements (u_r, u_z, u_θ). The three equilibrium equations and six strain-displacement equations in cylindrical coordinates appear in many standard texts and thus are not repeated here. However, for later use, we list the constitutive equations of thermoelasticity:

Normal stress-strain-temperature equations:

$$\sigma_r = \lambda\Theta_m + 2G(\varepsilon_r - \alpha_r T), \;\; \sigma_z = \lambda\Theta_m + 2G(\varepsilon_z - \alpha_r T),$$
$$\sigma_\theta = \lambda\Theta_m + 2G(\varepsilon_\theta - \alpha_r T) \tag{2.1}$$

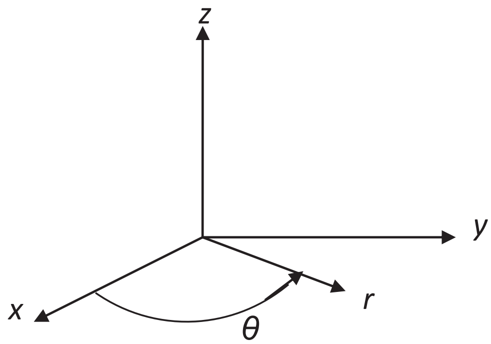

Fig. 1. Cylindrical coordinate system.

Shear stress-strain equations:

$$\tau_{rz} = G\gamma_{rz}, \quad \tau_{r\theta} = G\gamma_{r\theta}, \quad \tau_{z\theta} = G\gamma_{z\theta} \tag{2.2}$$

where Θ_m is the mechanical (stress-induced) dilatation, defined as

$$\Theta_m \equiv \Theta - 3\alpha_r T \tag{2.3}$$

Also, T is the temperature change (current temperature minus the stress-free temperature) and Θ is the dilatation (volume strain),

$$\Theta \equiv \varepsilon_r + \varepsilon_z + \varepsilon_\theta \tag{2.4}$$

The mean stress is,

$$\sigma_m \equiv \frac{\sigma_r + \sigma_\theta + \sigma_z}{3} \tag{2.5}$$

Additionally, λ and G are the Lame' elastic constants, where G is the shear modulus and α_r is the linear coefficient of thermal expansion of rubber. Additional useful relationships are those between various elastic constants:

$$\lambda = K - \tfrac{2}{3}G = \frac{E\nu}{(1+\nu)(1-2\nu)} \quad G = \frac{E}{2(1+\nu)} \tag{2.6}$$

where $K = \sigma_m/\Theta_m$ is bulk modulus, E is Young's modulus and ν is Poisson's ratio. For elastic behavior of rubber, $\lambda \cong K$.

Geometry and boundary conditions: Single and multilayer bearings are drawn in Fig. 2. The reinforcements (shims) are solid circular discs. They are assumed perfectly rigid with respect to mechanical loading; but they deform with uniform normal strains, $\alpha_S T$, where T is temperature change and α_S is the shim's linear coefficient of thermal expansion. The rubber pads are solid circular discs that are bonded to the shims; of course, a central hole could be included by using the singular solution.

The mechanical loading is the same as for the spherical bearing. There may be an all-around confining pressure p.

The simplicity of the *shearing and torsional displacement problems* enables direct derivation of accurate solutions without using the perturbation method. (This is similar to the shearing rotation and torsional rotation problems for the spherical bearing.) As a result, the equations in Sections 2.1.2 and 2.1.3 are not used with these two problems.

2.1.2. Selection of normalized variables

The same scheme as used for the spherical bearing is employed here. There are *two small normalized parameters* that will serve as the basis for using the perturbation method; the first, h, comes from the assumption that the rubber pad thickness, t_r, is thin relative to the pad diameter, D. The second,Δ, is based on the near-incompressibility of rubber, $G/\lambda \ll 1$. We select them as

$$h \equiv \frac{t_r}{D} \quad \text{and} \quad \Delta \equiv \sqrt{\frac{G}{\lambda}} \cong \sqrt{\frac{G}{K}} \tag{2.7}$$

Next, introduce a nondimensional radius,

$$\rho \equiv \frac{r}{R} \tag{2.8}$$

where $R = D/2$. For the perturbation method, all of the basic equations will be expressed in terms of the normalized radial coordinate

$$\bar{\rho} \equiv \frac{\Delta}{h}\rho \tag{2.9}$$

so that at the rubber free surface, $\bar{\rho} = \Delta/h$. A normalized axial coordinate is defined as

$$\bar{z} \equiv \frac{z}{Rh} = \frac{z}{t_r/2} \tag{2.10}$$

Thus, $\bar{z} = \pm 1$ at the upper and lower boundaries of the rubber pad in Fig. 2. The generic angular coordinate θ will be used without redefinition, with values expressed in radians.

The normalized stresses and displacements are defined like those for the spherical bearing, after accounting for the difference in the pad's thickness direction:

$$\bar{\sigma}_r \equiv \frac{h}{\lambda}\sigma_r, \ \bar{\sigma}_z \equiv \frac{h}{\lambda}\sigma_z, \ \bar{\sigma}_\theta \equiv \frac{h}{\lambda}\sigma_\theta, \ \bar{\tau}_{rz} \equiv \frac{h}{\lambda\Delta}\tau_{rz}, \ \bar{\tau}_{r\theta} \equiv \frac{h}{\lambda\Delta}\tau_{r\theta},$$
$$\bar{\tau}_{z\theta} \equiv \frac{h}{\lambda\Delta}\tau_{z\theta}, \ \bar{u}_r \equiv \frac{\Delta}{R}u_r, \ \bar{u}_z \equiv \frac{u_z}{R}, \ \bar{u}_\theta \equiv \frac{\Delta}{R}u_\theta \tag{2.11}$$

As in Part 1, the factor Δ is used so that all nine normalized variables have the same magnitude. Three additional normalized quantities and a dimensionless ratio are defined:

$$\bar{\Theta}_m \equiv h\Theta_m, \ \bar{T} \equiv \alpha_r hT, \ \bar{T}_S \equiv \alpha_S hT, \ k \equiv 3\frac{\Delta^2}{h^2} \tag{2.12}$$

Next, express the normalized stresses in terms of normalized displacements. This process provides the following equations:

$$\bar{\sigma}_r = \bar{\Theta}_m + 2\Delta^2\left(\frac{\partial \bar{u}_r}{\partial \bar{\rho}} - \bar{T}\right) \tag{2.13}$$

$$\bar{\sigma}_z = \bar{\Theta}_m + 2\Delta^2\left(\frac{\partial \bar{u}_z}{\partial \bar{z}} - \bar{T}\right) \tag{2.14}$$

$$\bar{\sigma}_\theta = \bar{\Theta}_m + 2\Delta^2\left(\frac{1}{\bar{\rho}}\frac{\partial \bar{u}_\theta}{\partial \theta} + \frac{\bar{u}_r}{\bar{\rho}} - \bar{T}\right) \tag{2.15}$$

$$\bar{\tau}_{rz} = \frac{\partial \bar{u}_r}{\partial \bar{z}} + \Delta^2\frac{\partial \bar{u}_z}{\partial \bar{\rho}} \tag{2.16}$$

$$\bar{\tau}_{r\theta} = \Delta\left(\frac{1}{\bar{\rho}}\frac{\partial \bar{u}_r}{\partial \theta} - \frac{1}{\bar{\rho}}\bar{u}_\theta + \frac{\partial \bar{u}_\theta}{\partial \bar{\rho}}\right) \tag{2.17}$$

$$\bar{\tau}_{z\theta} = \frac{\Delta^2}{\bar{\rho}}\frac{\partial \bar{u}_z}{\partial \theta} + \frac{\partial \bar{u}_\theta}{\partial \bar{z}} \tag{2.18}$$

2.1.3. Perturbation solution of equilibrium equations

Upon substitution of Eqs. (2.13)–(2.18) into the normalized version of the three equilibrium equations, we obtain three partial differential equations for the three normalized displacements. Only the zeroth order terms in each equation, for both Δ and h, are retained. Note that $\bar{\tau}_{r\theta}$ in Eq. (2.17) is automatically eliminated.

We find that the z-equilibrium equation reduces to

$$\frac{\partial \bar{\Theta}_m}{\partial \bar{z}} = 0 \tag{2.19}$$

Similar to the spherical bearing, we see Eqs. (2.13)–(2.15) imply the normal stresses are essentially equal to one another, and Eq. (2.19) shows the dilatation and normal stresses are essentially independent of the axial coordinate.

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