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Multi-directional behavior of granular materials and its relation to incremental elasto-plasticity

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ABSTRACT

The complex incremental behavior of granular materials is explored with multi-directional loading probes. An advanced discrete element model (DEM) was used to examine the reversible and irreversible strains for small loading probes, which follow an initial monotonic axisymmetric triaxial loading. The model used non-convex non-spherical particles and an exact implementation of the Hertz-like Cattaneo–Mindlin model for the contact interactions. Orthotropic true-triaxial probes were used in the study (i.e., no direct shear strain or principal stress rotation), with small strains increments of 2×10^{-6}. The reversible response was linear but exhibited a high degree of stiffness anisotropy. The irreversible behavior, however, departed in several respects from classical elasto-plasticity. A small amount of irreversible strain and contact slipping occurred for all directions of the stress increment (loading, unloading, transverse loading, etc.), demonstrating that an elastic domain, if it exists at all, is smaller than the strain increment used in the simulations. Irreversible strain occurred in directions tangent to the primary yield surface, and the direction of the irreversible strain varied with the direction of the stress increment. For stress increments within the deviatoric pi-plane, the irreversible response had rounded-corners, evidence of multiple plastic mechanisms. The response at these rounded corners varied in a continuous manner as a function of stress direction. The results are placed in the context of advanced elasto-plasticity models: multi-mechanism plasticity and tangential plasticity. Although these models are an improvement on conventional elasto-plasticity, they do not fully fit the simulation results.

1. Introduction

The mechanical behavior of granular materials is exceedingly complex, and engineers have been challenged for decades to measure, understand, and model this complexity. The current work reveals further complexity, using discrete element (DEM) simulations to improve current understanding of the multi-directional incremental yield and flow characteristics of a material that is initially loaded under monotonic triaxial conditions. Although several frameworks have been developed for structuring such experimental results, including elasto-plasticity (Yang et al., 2005), hypoplasticity (Wu et al., 1996; Lin et al., 2015), generalized plasticity (Hashiguchi, 2005), damage plasticity (Zhu et al., 2010), micro-mechanics-based homogenization (Nicot and Darve, 2011), endochronic (Yeh and Lin, 2006), and shock and fracture-based (Ganzenmüller et al., 2011) schema, we will place our results in the context of elasto-plasticity, which is currently the prevailing framework for rate-independent materials. Conventional elasto-plasticity is founded on six principles:

1. strain increments can be separated into distinct elastic and plastic parts as a sum or tensor product with each part being a homogeneous function of the stress increment, and with the elastic increment being a reversible function of the stress increment that is fully recovered after a closed loading–unloading cycle in stress-space;
2. the elastic increment is a linear function — both homogeneous and additive — of the stress increment;
3. the space of stress increments includes a finite elastic region within which no plastic deformation occurs;
4. the regions of elastic and plastic behavior are separated by a hyperplane (incremental yield surface) in stress-space;
5. plastic strain increments occur in a single direction, which can depend upon the current stress and its history but not upon the direction of the stress increment; and
6. the magnitude of the plastic strain is proportional to the projection of the stress increment onto the normal of the yield surface.

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Table 1
Summary of multi-directional probe studies with laboratory tests and numerical simulations.

Study	Shape	Contacts	Method[a]	Probe type[b]	$\|d\boldsymbol{\varepsilon}\|$ or $\|d\boldsymbol{\sigma}\|$
Lewin and Burland (1970)	shale powder	triaxial	–	–	10 kPa
Tatsuoka and Ishihara (1974)	sand	triaxial	–	E–P	0.001–0.006
Bardet (1994)	disks	linear	DEM	E–P	$\approx 1\times10^{-3}$
Anandarajah et al. (1995)	sand	–	triaxial	E–P	$\approx 1\times10^{-4}$
Royis and Doanh (1998)	sand	–	triaxial	E–P	$\approx 1\times10^{-4}$
Kishino (2003)	spheres	linear	GEM	E–P	1 kPa
Alonso-Marroquín and Herrmann (2005); Alonso-Marroquín et al. (2005)	polygons	linear	DEM	E–P	10 kPa $\approx 1\times10^{-5}$
Calvetti et al. (2003); Tamagnini et al. (2005)	spheres	linear[c]	DEM	R–I	$\approx 2\times10^{-4}$
Sibille et al. (2007)	spheres	linear	DEM	R–I	–
Plassiard et al. (2009)	spheres	linear[d]	DEM	R–I	$\approx 2\times10^{-5}$
Froiio and Roux (2010)	disks	linear	DEM[e]	–	$\approx 4\times10^{-6}$
Harthong and Wan (2013)	spheres	linear	DEM	E–P	0.1kPa
Wan and Pinheiro (2014)	spheres	–	DEM	R–I & E–P	0.1 kPa
Kuhn and Daouadji (2018)	sphere-clusters	linear & Hertz	DEM	R–I & E–P	2×10^{-6}
Current work	sphere-clusters	Hertz	DEM	R–I	2×10^{-6}

[a] GEM = Granular Element Method; triaxial = laboratory experiments.
[b] R–I = reversible-irreversible; E–P = elastic-plastic (see Section 2.2).
[c] With rotational restraint of particles.
[d] With rolling friction of contacts.
[e] Hybrid method: assembly loaded with DEM; probes applied with method similar to GEM (Kishino, 2003; Agnolin and Roux, 2007).

Table 2
Results of previous 2D simulations and their conformance with the six principles of conventional elasto-plasticity: Y = conforms with the principle, N = contradicts the principle.

Elasto-plasticity principle	Bardet (1994)	Alonso-Marroquín and Herrmann (2005); Alonso-Marroquín et al. (2005)	Froiio and Roux (2010)	Sibille et al. (2007)[a]	Plassiard et al. (2009)[a]
(1) $d\boldsymbol{\varepsilon} = d\boldsymbol{\varepsilon}^{(e)} + d\boldsymbol{\varepsilon}^{(p)}$, $d\boldsymbol{\varepsilon}^{(e)}$ is reversible		Y			
(2) $d\boldsymbol{\varepsilon}^{(e)}$ linear: $d\varepsilon_{ij}^{(e)} = C_{ijkl} d\sigma_{kl}$	Y	Y			Y
(3) Finite elastic domain		Y		Y	
(4) $d\boldsymbol{\varepsilon}^{(e)}$ & $d\boldsymbol{\varepsilon}^{(p)}$ domains are semi-spaces, normal **f**	Y	Y	Y	Y	Y
(5) Plastic increments $d\boldsymbol{\varepsilon}^{(p)}$ in single flow direction **g**	Y	Y	Y	Y	Y
(6) $\|d\boldsymbol{\varepsilon}^{(p)}\| = \mathbf{f}\cdot d\boldsymbol{\sigma}$	Y	Y	Y		Y

[a] Three-dimensional assemblies, but with axisymmetric triaxial probes.

Table 3
Results of previous 3D simulations and their conformance with the six principles of conventional elasto-plasticity: Y = conforms with the principle, N = contradicts the principle.

Elasto-plasticity principle	Kishino (2003)	Calvetti et al. (2003)	Tamagnini et al. (2005)	Harthong and Wan (2013)	Wan and Pinheiro (2014)
(1) $d\boldsymbol{\varepsilon} = d\boldsymbol{\varepsilon}^{(e)} + d\boldsymbol{\varepsilon}^{(p)}$, $d\boldsymbol{\varepsilon}^{(e)}$ is reversible					Y
(2) $d\boldsymbol{\varepsilon}^{(e)}$ linear: $d\varepsilon_{ij}^{(e)} = C_{ijkl} d\sigma_{kl}$	Y	Y			
(3) Finite elastic domain	Y	Y	N		
(4) $d\boldsymbol{\varepsilon}^{(e)}$ & $d\boldsymbol{\varepsilon}^{(p)}$ domains are semi-spaces, normal **f**	N	Y[a]		N	
(5) Plastic increments $d\boldsymbol{\varepsilon}^{(p)}$ in single flow direction **g**	N		N	N	N
(6) $\|d\boldsymbol{\varepsilon}^{(p)}\| = \mathbf{f}\cdot d\boldsymbol{\sigma}$	N	Y			

[a] "Y" applies to virgin loading conditions. A finite elastic domain was not found with pre-loaded conditions.

These principles have been tested with both laboratory experiments and simulations, in which soils or virtual assemblies of particles are loaded in small probes of stress or strain. Four laboratory programs and eleven simulation studies are summarized in Table 1. The simulation studies include the early two-dimensional (2D) studies of Bardet (1994) and Alonso-Marroquín et al. (2005) and more recent three-dimensional (3D) simulations using sphere assemblies. Although laboratory tests and simulations have exposed important aspects of granular behavior, disagreement or ambiguity still remains some the principles enumerated above. Tables 2 and 3 summarize simulation results for 2D and 3D probe studies. The 2D simulations in Table 2 are limited in the range of stress-space that can be accessed, and in this sense, they are similar to 3D axisymmetric triaxial conditions, which limit the accessible space to a two-dimensional hyperplane of the principal stress components, and the 3D studies also shown in the same table. This table shows that, when tested, each of the six elasto-plasticity principles is affirmed with 2D simulations, as indicated by the "Y" cells. Some behaviors were not tested in certain studies, and these cells are left blank. Conspicuous ambiguity arises, however, in the results of 3D simulations conducted with true-triaxial conditions, in which the increments of all three principal stresses were independently varied (Table 3). For example, Tamagnini et al. (2005) found that some plastic deformation, albeit small, occurred regardless of the direction of small loading probes, a result that is contrary to other 3D (and 2D) studies and to principle 3. In particular, they found that plastic strains occur for stress increments in opposite directions (i.e., for both loading and unloading), thus violating the third principle. Based upon their triaxial tests of sand,

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