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Interpolation Macdonald polynomials  
and Cauchy-type identitiesGrigori Olshanski <sup>a,b,c</sup><sup>a</sup> Institute for Information Transmission Problems of the Russian Academy  
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## ABSTRACT

Let  $\text{Sym}$  denote the algebra of symmetric functions and  $P_\mu(\cdot; q, t)$  and  $Q_\mu(\cdot; q, t)$  be the Macdonald symmetric functions (recall that they differ by scalar factors only). The  $(q, t)$ -Cauchy identity

$$\sum_{\mu} P_{\mu}(x_1, x_2, \dots; q, t) Q_{\mu}(y_1, y_2, \dots; q, t) = \prod_{i, j=1}^{\infty} \frac{(x_i y_j t; q)_{\infty}}{(x_i y_j; q)_{\infty}}$$

expresses the fact that the  $P_{\mu}(\cdot; q, t)$ 's form an orthogonal basis in  $\text{Sym}$  with respect to a special scalar product  $\langle \cdot, \cdot \rangle_{q, t}$ . The present paper deals with the inhomogeneous *interpolation* Macdonald symmetric functions

$$I_{\mu}(x_1, x_2, \dots; q, t) = P_{\mu}(x_1, x_2, \dots; q, t) + \text{lower degree terms.}$$

These functions come from the  $N$ -variate interpolation Macdonald polynomials, extensively studied in the 90s by Knop, Okounkov, and Sahi. The goal of the paper is to construct symmetric functions  $H_{\mu}(\cdot; q, t)$  with the biorthogonality property

$$\langle I_{\mu}(\cdot; q, t), H_{\nu}(\cdot; q, t) \rangle_{q, t} = \delta_{\mu\nu}.$$

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These new functions live in a natural completion  $\widehat{\text{Sym}} \supset \text{Sym}$ . As a corollary one obtains a new Cauchy-type identity in which the interpolation Macdonald polynomials are paired with certain multivariate rational symmetric functions. The degeneration of this identity in the Jack limit  $(q, t) = (q, q^{\mathbf{k}}) \rightarrow (1, 1)$  is also described.

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**Contents**

1.	Introduction . . . . .	66
1.1.	Preface . . . . .	66
1.2.	The results . . . . .	68
1.3.	Examples and comments . . . . .	69
1.4.	Organization of the paper . . . . .	70
1.5.	Acknowledgments . . . . .	70
2.	Interpolation polynomials and functions . . . . .	71
2.1.	Preliminaries . . . . .	71
2.2.	Interpolation Macdonald polynomials . . . . .	72
2.3.	Interpolation symmetric functions . . . . .	74
3.	Dual interpolation functions . . . . .	78
3.1.	Biorthogonal systems . . . . .	78
3.2.	The algebra $\widehat{\text{Sym}}_{\mathbb{F}}$ and the dual functions $H_{\mu}(\cdot; q, t)$ . . . . .	79
3.3.	The Cauchy identity . . . . .	80
3.4.	Restriction to $N$ variables . . . . .	80
3.5.	Modified dual functions . . . . .	81
4.	The case $t = q$ . . . . .	82
5.	Combinatorial formula . . . . .	87
5.1.	Formulation of the theorem . . . . .	87
5.2.	A reformulation . . . . .	88
5.3.	Proof of Theorem 5.4: reduction to a Pieri-type formula . . . . .	89
5.4.	Proof of Theorem 5.4: computation of Pieri coefficients . . . . .	90
6.	Difference operators: special case $t = q$ . . . . .	97
7.	Difference operators: general case . . . . .	99
7.1.	$q$ -Difference equations for polynomials $I_{\mu N}(\cdot; q, t)$ . . . . .	99
7.2.	$q$ -Difference equations for modified dual functions $\tilde{H}_{\mu N}(\cdot; q, t)$ . . . . .	102
8.	The Jack limit . . . . .	104
9.	Specializations at $t = 0$ and $q = 0$ . . . . .	112
9.1.	The case $t = 0$ . . . . .	112
9.2.	The case $q = 0$ . . . . .	113
	References . . . . .	116

**1. Introduction**

*1.1. Preface*

One of the fundamental formulas in the theory of symmetric functions is the Cauchy identity

$$\sum_{\mu \in \mathbb{Y}} s_{\mu}(x_1, x_2, \dots) s_{\mu}(y_1, y_2, \dots) = \prod_{i=1}^{\infty} \prod_{j=1}^{\infty} \frac{1}{1 - x_i y_j}.$$

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