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Interpolation Macdonald polynomials and Cauchy-type identities

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ABSTRACT

Let Sym denote the algebra of symmetric functions and $P_\mu(\,\cdot\,;q,t)$ and $Q_\mu(\,\cdot\,;q,t)$ be the Macdonald symmetric functions (recall that they differ by scalar factors only). The (q,t)-Cauchy identity

$$\sum_\mu P_\mu(x_1,x_2,\dots;q,t)Q_\mu(y_1,y_2,\dots;q,t)=\prod_{i,j=1}^{\infty}\frac{(x_iy_jt;q)_\infty}{(x_iy_j;q)_\infty}$$

expresses the fact that the $P_\mu(\,\cdot\,;q,t)$'s form an orthogonal basis in Sym with respect to a special scalar product $\langle\,\cdot\,,\,\cdot\,\rangle_{q,t}$. The present paper deals with the inhomogeneous *interpolation* Macdonald symmetric functions

$$I_\mu(x_1,x_2,\dots;q,t)=P_\mu(x_1,x_2,\dots;q,t)+\text{lower degree terms.}$$

These functions come from the N-variate interpolation Macdonald polynomials, extensively studied in the 90s by Knop, Okounkov, and Sahi. The goal of the paper is to construct symmetric functions $H_\mu(\,\cdot\,;q,t)$ with the biorthogonality property

$$\langle I_\mu(\,\cdot\,;q,t),H_\nu(\,\cdot\,;q,t)\rangle_{q,t}=\delta_{\mu\nu}.$$

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These new functions live in a natural completion $\widehat{\mathrm{Sym}} \supset \mathrm{Sym}$. As a corollary one obtains a new Cauchy-type identity in which the interpolation Macdonald polynomials are paired with certain multivariate rational symmetric functions. The degeneration of this identity in the Jack limit $(q,t) = (q, q^{\mathbf{k}}) \to (1,1)$ is also described.

Contents

1. Introduction

1.1. Preface

One of the fundamental formulas in the theory of symmetric functions is the Cauchy identity

$$\sum_{\mu \in \mathbb{Y}} s_\mu(x_1, x_2, \dots) s_\mu(y_1, y_2, \dots) = \prod_{i=1}^{\infty} \prod_{j=1}^{\infty} \frac{1}{1 - x_i y_j}.$$