Annals of Nuclear Energy 124 (2019) 49–57

Contents lists available at ScienceDirect

Annals of Nuclear Energy

journal homepage: www.elsevier.com/locate/anucene

Inverse estimation methods of unknown radioactive source for fuel debris search

Shinji Sugaya, Tomohiro Endo, Akio Yamamoto *

Nagoya University, Furo-cho, Chikusa-ku, Nagoya, Aichi 464-8603, Japan

ARTICLE INFO

Article history:
Received 26 June 2018
Received in revised form 31 August 2018
Accepted 14 September 2018

Keywords:
Inverse estimation
Fuel debris
Fukushima Daiichi Nuclear Power Plant
Adjoint transport calculation
Detection probability

ABSTRACT

To identify the distribution of fuel debris remaining in the reactor vessel and/or the containment vessel of Fukushima Daiichi NPS, we focused on the inverse estimation of radioactive source distribution using the measured radiation counts. The Maximum Likelihood-Expectation Maximization (ML-EM) and the Moore-Penrose Matrix Inverse (MPMI) methods are examined. The ML-EM method has been used for the image reconstruction of computed tomography, and the MPMI method is one of the solution methods for simultaneous linear equations with the underdetermined condition. A simple calculation model simulating a containment vessel was constructed including detectors and radiation sources. In an actual situation, a sufficient number of radiation measurement positions would not be available owing to the complexity of structures inside the containment vessel. Thus, the number of radiation measurement points (number of constraints) is smaller than that of radiation source positions. It means that an underdetermined inverse problem should be solved. The detection probability of radiation (neutron or photon) is calculated by the adjoint transport calculation since the detection probability is used as the coupling coefficient between radiation counts at a detector and a radioactive source. The result of estimation using the ML-EM or the MPMI method indicates that the accuracy of estimation depends on the distance between a radiation source and a detector, and measurement positions of radiation count. The ML-EM and the MPMI methods show different prediction accuracy depending on the prediction condition. It is found that reasonable prediction accuracy would be obtained when the detectors are placed at the vicinity of radiation sources of interest.

1. Introduction

In Fukushima Daiichi Nuclear Power Plant (1F), decommissioning of severely damaged nuclear reactors is being carried out. The retrieval of fuel debris remaining in the reactor pressure vessel and/or the containment vessel (RPV/CV) is expected as one of the most difficult tasks since we have few past experiences of similar works. Information on the distribution of fuel debris in RPV/CV is essential to establish a roadmap to remove it, to prevent an unexpected re-critical accident during removal works, and to determine radiation shielding from fuel debris during removal works. Especially the risk of re-criticality of fuel debris during removal works is studied in Tonoike et al. (2015) and so on, and it is one of the potential issues in the decommissioning of 1F.

Investigation of fuel debris using muon suggested that most of the fuel debris would not exist in RPV of Unit 1. In Unit 3, a robot was sent to the inside of the CV in July 2017 and something (possibly fuel debris) once melted and then frozen was found at the bottom of the CV. Contrary, in Unit 2, most of the fuel debris would still exist at the RPV bottom head (Tokyo Electric Power Company, 2016). However, more detail estimation of fuel debris distribution is highly desirable to establish a removal plan of fuel debris. Location identification of fuel debris is being tried, e.g., in Katakura et al. (2016), but the improvement of reliability of prediction results is desirable using various approaches.

In this study, we try to estimate the distribution of radiation sources using multiple measurement results of radiation and inverse analysis methods, i.e., the Maximum Likelihood-Expectation Maximization (ML-EM) method (Shepp and Valdi, 1982) and the Moore-Penrose Matrix Inverse (MPMI) method (Ben-Israel and Greville, 2003). In these inverse analysis methods, detection probability of a radiation emitted from a radiation source is necessary. Though various approaches can be used to evaluate the detection probability, an adjoint transport calculation using the discrete ordinate method is adopted. The idea of using detector counts for the estimation of radiation sources is similar to the

* Corresponding author.
E-mail addresses: s-sugaya@fermi.nucl.nagoya-u.ac.jp (S. Sugaya), t-endo@energy.nagoya-u.ac.jp (T. Endo), a-yamamoto@energy.nagoya-u.ac.jp (A. Yamamoto).

https://doi.org/10.1016/j.anucene.2018.09.022
0306-4549/

study in Shikaze et al. (2016). A simplified calculation model of the CV is constructed and used for analysis. Through verification calculations, the effectiveness and validity of the present method are confirmed.

In Section 2, the theories of inverse estimation methods are described. The calculation procedures and results are described in Sections 3 and 4, respectively. Finally, concluding remarks are summarized in Section 5.

2. Theory

The structures inside RPV/CV are complicated as well as the distribution of fuel debris. On the other hand, measurement points of radiation inside RPV/CV will be very limited. Therefore, the inverse problem of this study should treat an underdetermined system, where the number of unknowns (radioactivity) is larger than that of constraint conditions (radiation measurement results by detectors). Note that distribution of actual radioactivity would be spatially continuous but it is spatially discretized in the inverse estimation. Therefore, number of unknowns depends on spatial discretization, i.e., number of unknowns becomes larger or smaller with detail or coarse discretization, respectively.

There are some techniques to estimate a plausible solution for such an inverse problem. In this study, we use the following two methods: One is the ML-EM method; another is the MPMI method to conduct inverse estimation. In the present study, the original three-dimensional geometry is approximated by the two-dimensional R-Z geometry.

There are various sources of uncertainty that affect the inverse estimation results and they are discussed in Section 2.3. However, the uncertainties are not explicitly taken into account in the present study since the purpose of the present study is the proof of principle using simplified conditions.

2.1. Maximum Likelihood-Expectation Maximization (ML-EM) method

The ML-EM method is one of the inverse estimation techniques based on the Bayesian theory. The ML-EM method is effectively utilized in the image reconstruction of computed tomography (CT) (Słomski et al., 2014; Parra and Barrett, 1998). The inverse problem of this study is similar to that of CT, thus we try applying the ML-EM method to estimate radioactivity distribution. There are other inverse estimation methods (Parra and Barrett, 1998; Tsui et al., 1991), but they are not used in this study except for the ML-EM and the MPMI methods.

As previously described, the radioactivity distribution is spatially discretized and treated as the discretized point sources whose number is J. The radioactivity can be estimated by a probable radioactivity at point j (A_j) and the measured radiation count of detector i (y_i). In the ML-EM method, radioactivity is updated with an iterative procedure using the probable radioactivities obtained by the previous iteration or by the initial guess. Using Eq. (1), the estimated radioactivity distribution can be finally obtained as the converged solution:

$$A_j^{k+1} = \frac{A_j^k}{\sum_{i=1}^{I} C_{ij}} \sum_{i=1}^{I} \frac{y_i C_{ij}}{\sum_{j=1}^{J} C_{ij} A_j^k}, \tag{1}$$

where notations are defined as follows; A: radioactivity of point radiation source, y: radiation count of detector, i: detector index (from 1 to I), j: radiation source index (from 1 to J), C_{ij}: detection probability of a radiation at detector (i) emitted from radiation source (j), k: number of iterations in the ML-EM method.

Eq. (1) is derived using the Poisson distribution and the Bayes' theorem. The ML-EM method has the property of Bayesian estimation that provides more plausible estimation from the observation results and the probable estimation that comes from the previous iteration of the initial guess.

2.2. Moore-Penrose matrix inverse (MPMI) method

The inverse problem for radioactivity in this study can be expressed by the following simultaneous linear equations:

$$\mathbf{C}\vec{A} \approx \vec{y}, \tag{2}$$

where $\mathbf{C}$ is a $I \times J$ matrix of detection probabilities, I and J are the numbers of detectors and radiation source points, respectively, $\vec{A}$ is the spatially discretized radiation source, $\vec{y}$ is the measured radiation count at each detector. For example, Eq. (2) is rewritten as Eq. (3) when the numbers of detectors and radiation sources are 2 and 3, respectively:

$$\begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} \approx \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}. \tag{3}$$

Fig. 1 describes this situation.

Simultaneous linear equations of an underdetermined system cannot be solved by a normal inverse matrix. However, even in this case, the minimum L2-norm solution $\vec{A}_{MPMI}$ can be numerically solved using the MPMI method (Wolfram, 2017; Chen et al., 2017):

$$\vec{A}_{MPMI} = \begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \end{pmatrix}^{+} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \tag{4}$$

where the superscript + means the Moore-Penrose pseudoinverse. The L2-norm minimization condition is necessary to uniquely obtain the solution of $\vec{A}$ for an underdetermined system. For example, when the numbers of constraint conditions and unknowns are respectively 2 and 3, all solutions of (A_1, A_2, A_3) satisfying Eq. (2) correspond to arbitrary points on the plane-plane intersection, i.e., $C_{11}A_1 + C_{12}A_2 + C_{13}A_3 = y_1$ and $C_{21}A_1 + C_{22}A_2 + C_{23}A_3 = y_2$. The minimum L2-norm solution $(A_{MP,1}, A_{MP,2}, A_{MP,3})$ corresponds to the nearest point from the origin $(0, 0, 0)$ among the intersection. In the inverse estimation of radiation source distribution, the minimization of the L2-norm is applied to the spatially discretized radiation source, $\vec{y}$. Therefore, as discussed in Section 3, the elements in $\vec{y}$ (the discretized point radiation sources) tend to become as small as possible while satisfying Eq. (2).

Numbers of radiations counts (y_i) and the discretized radiation sources (A_j) used in realistic calculations is larger than those of the above toy problem (2 and 3, respectively). For example, numbers of radiations counts (y_i) and the discretized radiation sources (A_j) are 12 and 128, respectively, in the numerical calculations in Section 4. Extension to such realistic condition is straightforward, i.e., just increase the numbers of detectors and the discretized radiation sources in Eq. (2). Note that geometrical modeling is independent from the calculation procedures of the ML-EM or the MPMI method since geometrical positions of detectors or radiation sources do not directly appear in Eq. (2).

In order to numerically solve $\vec{A}_{MPMI}$, the singular value decomposition is used. We can perform the singular value decomposition by the proper matrices of $\mathbf{U}, \mathbf{V}$, and the singular matrix of $\mathbf{C}$:

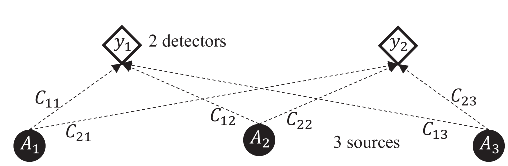

Fig. 1. Three radiation sources and two detectors.