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Semiparametric inference for an extended geometric failure rate reduction model

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ABSTRACT

The aim of this paper is twofold. First a new imperfect maintenance model is introduced. This model is an extension of Finkelstein's Geometric Failure Rate Reduction model, using the modification proposed by Bordes and Mercier for extending the Geometric Process. Second, based on the observation of several systems, the semiparametric inference in this model is studied. Estimators of the euclidean and functional model parameters are derived and their asymptotic normality is proved. A simulation study is carried out to assess the behavior of these estimators for samples of small or moderate size. Finally, an application on a real dataset is presented.

1. Introduction

The basic assumptions on maintenance effectiveness for the failure process of repairable systems are known as minimal maintenance (or As Bad As Old effect) and perfect maintenance (or As Good As New effect). In the minimal maintenance case, each repair leaves the system in the same state as it was before failure and the obtained process is a nonhomogeneous Poisson process (NHPP). In the perfect maintenance case, each repair is perfect and leaves the system as if it were new, the obtained process is then a renewal process (RP). In practice, it is well known that the reality is between these two extreme cases. A standard maintenance action is better than minimal but not necessarily perfect. This leads to the notion of imperfect maintenance for repairable systems.

Many imperfect maintenance models have been proposed. Some of the most famous models are the model of Brown and Proschan (1983) (BP), Kijima's virtual age models (Kijima, 1989) and the Geometric Process (GP) model (Lam, 1988). An extension of this last model has been recently proposed by Bordes and Mercier (2013). Kijima's models of types I and II have been modified by Baxter et al. (1996) for more realistic interpretations. Classes of imperfect repair models based on an arithmetic reduction of failure intensity (ARI) or an arithmetic reduction of virtual age (ARA) have been proposed by Doyen and Gaudoin (2004). The ARA and ARI models are very flexible and have been widely used. The idea of using a geometric reduction instead of an arithmetic reduction has been proposed in Doyen and Gaudoin (2004) but has been deepened only

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very recently in Doyen et al. (2017). The first geometric reduction of intensity model has been introduced by Finkelstein (2008) under the name of Geometric Failure Rate Reduction (GFRR) model. Since the GFRR model suffers from the same drawbacks as the GP model, the first contribution of this paper is to extend the GFRR model, using the idea proposed by Bordes and Mercier (2013) for extending the GP model.

For aging systems undergoing imperfect corrective maintenances (CM) after failures, it is important to be able to improve system reliability by performing preventive maintenances (PM, see for instance Liao et al. (2010)). A usual sequential maintenance policy consists in replacing the system by a new one after N maintenances. This policy can be found for instance in Nakagawa (1988) and Zhang et al. (2013). In these papers, all the model parameters are considered to be known and the objective is to determine an optimal value of N. In the present paper, we assume that a more general sequential maintenance policy is used and our aim is to estimate the model parameters.

It has to be noted that the question of statistical inference is not frequently treated in the literature on imperfect maintenance. Basically, the parameters are estimated using a fully parametric maximum likelihood approach. Some exceptions are Lam (1992), Peña and Hollander (2004) and Beutner et al. (2017). The first one studied non-parametric inference in the GP model. The second one introduced a general model which takes into account covariates, factors of heterogeneity and virtual age. The semiparametric inference for this model has been studied by Peña et al. (2007) when the virtual ages are supposed to be known. The last one studied the semiparametric inference for ARA models. In this paper, we study the semiparametric inference for the extended GFRR model.

The remainder of this paper is organized as follows. Section 2 briefly reviews some existing imperfect maintenance models involving a reduction of age or intensity. Section 3 introduces the new extended GFRR model with the proposed sequential maintenance policy. Section 4 develops the estimation procedure of the euclidean and functional parameters of this model. The study of their asymptotic behaviors is done in Section 5. Experimental results based on simulated data are presented in Section 6. In Section 7, the model is applied to the Norsk Hydro ammonia plant dataset introduced by Bunea et al. (2003). Finally, Section 8 draws some conclusions and prospects.

2. Some imperfect maintenance models

Let us consider a repairable system with successive failure times denoted by:

$$0 = T_0 < T_1 < T_2 < \cdots < T_k < \cdots .$$

After each failure, a repair, or corrective maintenance (CM), is performed. Repair durations are assumed to be negligible, so the failure and repair times can be confounded.

Let $X_k = T_k - T_{k-1}$, for $k \in \mathbb{N}^*$, be the interfailure times and let $\lambda_{X_k}(\cdot)$ denote the instantaneous hazard rate function of X_k. Let $(N_t)_{t\in\mathbb{R}^+}$ be the associated counting process of the number of failures observed up to time t. The failure intensity of this process is defined by:

$$\lambda_t = \lim_{\Delta t\to 0} \frac{1}{\Delta t} P(N_{(t+\Delta t)^-} - N_{t^-} = 1|\mathcal{H}_{t^-}).$$

Many papers have considered the problem of modeling the effect of maintenance on the reliability of a system. In order to see how our model takes place in the literature, let us review a part of the existing models with a unified writing in terms of their failure intensity, even if they might have been introduced differently. As said above, the simplest models are the As Good As New (AGAN) and As Bad As Old (ABAO) ones. In the first case, the repair is perfect and the failure intensity is:

$$\lambda_t = \lambda_{X_1}(t - T_{N_{t^-}}),$$

leading to a renewal process. When a minimal repair is operated, in the ABAO case, the counting process $(N_t)_{t\in\mathbb{R}^+}$ is a non-homogeneous Poisson process with failure intensity

$$\lambda_t = \lambda_{X_1}(t).$$

Since the reality is generally between these two extreme cases, imperfect maintenance models have been introduced, including the BP model, the Geometric Processes, the Virtual Age models as well as the models of reduction of virtual age or reduction of intensity. The Arithmetic Reduction of Age model with memory 1, introduced by Doyen and Gaudoin (2004) and denoted ARA_1, has a failure intensity of the form

$$\lambda_t = \lambda_{X_1}(t - \rho T_{N_{t^-}}),$$

where ρ characterizes the effect of maintenance. The AGAN and ABAO models can be obtained with respectively the special cases $\rho = 1$ and $\rho = 0$. Imperfect maintenance corresponds to $0 < \rho < 1$. The ARA_1 model appears to be a Kijima's type I model (see Kijima (1989)) with deterministic repair effects. Kijima's type II model (see again Kijima (1989)) with deterministic repair effects leads to the Arithmetic Reduction of Age model with infinite memory or ARA_∞ introduced by Doyen and Gaudoin (2004). The failure intensity is in this case

$$\lambda_t = \lambda_{X_1}\left(t - \rho \sum_{j=0}^{N_{t^-}-1} (1-\rho)^j T_{N_{t^-}-j}\right).$$