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Original articles

Moments and distributions of the last exit times for a class of Markov processes

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Abstract

In this paper, we consider some related questions to last exit time of general Markov processes. An estimate for the distribution of the last exit time is derived. An equivalent characterization for the finiteness of the k-moments of the last exit time is obtained. Finally, some examples are provided to show the significance and usefulness of our results.

Keywords: Markov process; Last exit time; Moment; Distribution

1. Introduction

Let $X = (\Omega, \mathcal{F}, \mathcal{F}_t, X_t, \theta_t, P^x)$ be a Hunt process with a general state space $(E, \mathcal{E})$. For any $B \subset E$, define $L_B = \sup\{t \geq 0 : X_t \in B\}$ and we call L_B the last exit time from B of X. Due to its wide application in potential theory, the research of the last exit times has attracted an increasing amount of attention. The readers can refer to Chung [4], Getoor [5] and the references therein. Besides, as an important tool, Markov processes theory is widely used in other fields, such as stochastic neural networks, Markovian switching system and so on. For these topics, we can refer to [1,13,18,24] and the references therein.

An important question is how to compute the distribution of last exit time. Due to the last exit time is not a stopping time, it is hard to derive the precise expression for it. Pitman [12] and Getoor [6] showed that if X is the standard Brownian motion in $\mathbb{R}^d (d \geq 3)$, then $L_{B(0,r)}$ has a density given by

$$r^{d-2}\left[2^{\frac{d-2}{2}}\Gamma\left(\frac{d-2}{2}\right)t^{\frac{d}{2}}\right]^{-1} e^{-\frac{r^2}{2t}}.$$

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For more general Markov processes, Chen [3] gave the density of last exit time for Bessel processes. Wang [21,20] and Li et al. [9] studied the last exit distributions for several special classes of Markov processes. Li and Liu [8] derived the estimate of last exit time for elliptic diffusion processes. For more general Markov processes, it is very difficult to get the precise expression for the last exit distributions.

An important application for last exit time is to research the transience and recurrence for Markov processes. We call X is transient if $L_K < \infty$, a.s. for any compact set K. Furthermore, X is called strongly transient if $E^x(L_B^k) < \infty$ for any compact set K. For the transient and the strongly transient criterion, there are some results in the previous literature. For example, Sato et al. [16,15], Shiga [17] considered the transience and recurrence criterion for Ornstein–Uhlenbeck type Markov processes. Sato [14] derived a criterion of strongly transient for a class of Lévy process by using characteristic functions. Yamamuro [23] obtained a criterion of strongly transient for Ornstein–Uhlenbeck type Markov processes. The finiteness of the high moments of last exit time is also an important question, i.e. when does $E^x(L_B^k) < \infty$ for $k \geq 1$ and all compact set B. Takeuchi [19] showed that if X is a α symmetric stable process taking values in $\mathbb{R}^d$, then $E^x(L_B^k) < \infty$ for $k \geq 1$ and all compact set B if and only if $\alpha(k+1) < d$. Hawkes [7] generalized the results to the case of symmetric Lévy processes, and derived that the high moments of the last exit time is finite if and only if $\int_{|z|\leq 1}[\psi(z)]^{-(k+1)} < \infty$. Here $\psi(z)$ is the characteristic exponent for Lévy processes. Li et al. [8] studied the finiteness of the higher moments of the last exit time for elliptic diffusion processes taking values in $\mathbb{R}^d$. They proved that $E^x(L_B^k) < \infty$ for $k \geq 1$ and all compact set B if and only if $d \geq 5$. For more general Markov processes, it is hard to research the above question.

All the results in the above focus on well-property Markov processes. It is natural to ask the following questions: (1) Can one provide useful estimates for the distribution of L_B for general Markov processes? (2) Can one obtain a sufficient and necessary condition for $E^x(L_B^k) < \infty$ ($k \geq 1$) for general Markov processes? Inspire of the discussion as above, we will consider these two questions for general Markov processes. By using the probability potential theory, we derive an estimate of distribution for last exit time and a sufficient and necessary condition for the existence of moments for last exit time. Finally, we provide some examples to show the significance and usefulness of our results.

2. Preliminaries and notations

Unless otherwise specified, we will use the notation and terminology adopted in Blumenthal and Getoor [2].

For any Borel set B, denote the last exit time from B: $L_B = \sup\{t \geq 0 : X_t \in B\}$ and the first hitting time of B: $T_B = \inf\{t > 0 : X_t \in B\}$. ($\sup \emptyset \equiv 0$ and $\inf \emptyset \equiv \infty$). We have the following relationship between L_B and T_B:

$$\{T_B \circ \theta_t < \infty\} = \{L_B > t\}$$

where θ_t is the shifting operator.

Let $P(t, x, B) = P^x(X_t \in A)$ be the transition function of X and $P_t f(x) \equiv \int_E P(t, x, dy) f(y)$ called the transition operator. $Uf(x) = E^x\left(\int_0^\infty f(X_t)dt\right)$ denotes the potential of nonnegative measurable function f. Furthermore, define $U^\lambda f(x) \equiv \int_0^\infty e^{-\lambda t} P_t f(x) dt$. $P_B f(x) \equiv E^x(f(X_{T_B}); T_B < \infty)$.

We recall the following definitions about transient and strongly transient. X is transient if

$$L_B < \infty, \quad P^x\text{-}a.s.\ \forall x \in E, \quad \forall\, compact\ set\ B.$$

This definition is equivalent to

$$P^x(\lim_{t\to\infty} |X_t| = \infty) = 1, \quad for\ every\ x \in E.$$

X is strongly transient if

$$E^x(L_B) < \infty, \quad P^x\text{-}a.s.\ \forall x \in E, \quad \forall\, compact\ set\ B.$$

Last, we will introduce the definition of Feller processes and weak Feller processes.

A Markov process is called a Feller process, if for any continuous bounded function f, $P_t f$ is still a continuous bounded function. A Markov process is called a weak Feller process, if for any continuous bounded function f, $U^\lambda f$ is a lower semi-continuous function.

We need the following notations. Γ° and $\overline{\Gamma}$ denote the interior and closure of set Γ. $suppf = \overline{\{x : f(x) \neq 0\}}$. $B(0, r) = \{x \in \mathbb{R}^d : |x| \leq r\}$. Sometimes, we will denote $L_{B(0,r)}$ by L_r for the sake of brief. $\mathcal{C}_c^+$ denote all the nonnegative continuous function with a compact support.